New Shell Closure in Exotic Neutron-rich Sn Isotope : Role of 3-Body Force

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The variation of excitation energy of the first 2^+ state ($E(2_1^+)$) of even-even (e-e) $^{134-140}$Sn has been calculated using shell model with empirical SMPN and realistic CWG interactions in the $\pi(gdsh) \otimes \nu(hfpi)$ valence space above ^{132}Sn core. The prediction of SMPN differs dramatically from that with CWG. It predicts decreasing $E(2_1^+)$ with increasing valence neutron number (ν) for $^{134-138}$Sn and a strong shell closure at ^{140}Sn. Spin-tensor decomposition of the interactions reveals that the origin of these features might lie in the three-body effects. Calculations with CWG3M interaction, which is obtained by including a simple 3-body monopole term in the CWG predict results similar to SMPN. The consequences of modification of the empirical SMPN interaction in the light of new mass measurements and possible ways of estimating 3 - body effects for this mass region have also been discussed.

1. Introduction

Shell evolution with increasing neutron number away from stability has been an issue of intense experimental and theoretical investigations in the recent times. Observations leading to modifications in the age-old neutron and proton magic numbers have generated great excitement in the nuclear physics community. Most of these observations and theoreical studies are centered around the lighter mass nuclei in the *sd* and *pf* shells. For heavier mass nuclei, so far no such experimental signature of washing away of shell closures and evolution of new ones have been reported. This is due to the difficulty in obtaining experimental information of nuclei too far from stability for these mass regions, unlike that in lighter Z nuclei. But there have beeni theoretical efforts to study the shell evolution for heavier nuclei also.

There are ten naturally abundant isotopes of Sn lying on stability line - $^{112,114-120,122,124}$Sn and the most neutron-rich among them is ^{124}Sn ($N/Z = 1.48$). Thus it is seen that nuclei in the ^{132}Sn ($N/Z = 1.64$) region are neutron-rich and lie at least 8 neutrons away from the line of stability. The studies in this closer-to-neutron-drip-line region above the doubly magic ^{132}Sn nucleus have recently revealed many intriguing issues concerning some newer aspects of nuclear structure and N-N interaction in such exotic environments. These nuclei with $50 \leq Z \leq 56$ and $82 \leq N \leq 90$ lie on or close to the path of the astrophysical r-process flow. So extensive knowledge about their structure, particularly the binding energies, low-lying excited states, and beta - decay rates at finite temperatures, are important ingredients for astrophysical calculations. Sn isotopes are of particular importance in this respect. However, many of these important nuclei are yet to be studied experimentally. Some spectroscopic information exists only for ^{134}Sn. Recently, the beta -decay half-lives via delayed neutron emission have been measured for ^{135}Sn to ^{137}Sn nuclei. These exotic Sn isotopes are characterised by fast forbidden β^- -decay from their ground states and therefore furnish valuable information on nuclear weak interaction process. Their small production rates and lifetimes are the main causes of the severe limitations in acquiring spectroscopic information on them. Therefore, reliable theoretical calculations and their comparisons with the

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hitherto available experimental data are not only important for extending our knowledge of N-N interaction and nuclear structure in the exotic neutron-rich environment, but they also provide very useful ingredients for reviewing the problems related to the nuclear astrophysics in general. It has been shown [1] that at thermal environment, the population of the probable low-lying 2_1^+ states in even-even Sn isotopes above ^{132}Sn which are connected to the (Sb) daughter states by allowed Gamow-Teller transitions can effectively increase their astrophysical half-lives compared to that for the fast forbidden decays from their respective ground states.

In this background, we present in this theoretical work, some new features and shell evolution in the neutron-rich semi - magic Sn isotopes above the doubly magic ^{132}Sn core. Our analysis of the Hamiltonians used for studying nuclear structure in the shell model formalism also points to the fact that the 3-body force might have a role in the shell evolution in this region too. We have also undertaken an effort to review our predictions in the light of newly measured mass data. Possible ways of estimating 3 - body effects for this mass region have also been discussed.

2. Formalism: model space and modified Hamiltonian

We have made an empirical interaction SMPN [2] by modifying CW5082 [3, 4] nuclear Hamiltonian derived from Kuo-Herling interaction of the ^{208}Pb region by tuning only a few of the matrix elements to reproduce the experimental data for two nucleon systems above ^{132}Sn inert core. The valence space consists of $\pi(1g_{7/2}, 2d_{5/2}, 2d_{3/2}, 3s_{1/2}, 1h_{11/2})$ and $\nu(1h_{9/2}, 2f_{7/2}, 2f_{5/2}, 3p_{3/2}, 3p_{1/2}, 1i_{13/2})$ orbitals. The motivation for construction of this new empirical Hamiltonian, SMPN, and the details of changes made are given in detail in Ref. [2] and the changed tbmes have been tabulated here in Table I (also see [5]).

In the present work, along with the empirical SMPN, we have used CWG, the realistic interaction obtained starting with a G matrix derived from the CD-Bonn nucleon-nucleon potential using the Q-box method [6]. In the Hamiltonians the same set of single-particle energies [7] of the valence orbitals but different sets of two-body interaction matrix elements (tbmes) have been used. These interactions reproduce the experimental excitation spectra and wave function properties such as the electromagnetic transition probabilities of neutron - rich nuclei in this mass region ($50 \leq Z \leq 56$, $82 \leq N \leq 88$) remarkably well [2, 7]. The shell model codes OXBASH and NUSHELL@MSU have been used [8]. The variation of excitation energy of the first 2^+ state ($E(2_1^+)$) of even-even (e-e) $^{134-140}$Sn [7, 9] has been calculated.

3. Results and Discussions

The first remarkable experimental observation is that the excitation energy of the first 2^+ state in ^{134}Sn is arround 730 keV much depressed than that observed ($E((2_1^+)$ is almost constant $\simeq$ 1200 keV) for A =102-130, Sn isotopes below the double shell closure at ^{132}Sn. The realistic CWG interaction predicts well the ($E(2_1^+)$) energy of ^{134}Sn.Of course both the interactions predict more or less similar spectra [9] for all the neutron rich nuclei in this mass region for which the experimental data are available. However their predictions differ dramatically for the experimentally unknown neutron rich isotopes of Sn, viz. ^{136,138}Sn [7, 9]. The CWG predicts nearly constant $E(2_1^+)$ for $^{134-138}$Sn isotopes, normally expected for singly-magic nuclei, similar to those below N = 82, for Sn isotopes with A=102-130, (Talmi's classical example) and a weak shell closure at ^{140}Sn. The Generalised seniority scheme predicts constant first 2_1^+ excitation energy along a given isotopic/isotonic chain [10]. On the other hand, SMPN predicts decreasing $E(2_1^+)$ with increasing valence neutron number (n) for $^{134-138}$Sn and a strong shell closure at ^{140}Sn. This decreasing trend fits [7] in the systematics of the experimental $E(2_1^+)$ of their isotones with Z>50. This also matches the systematics of $E(2_1^+)$ differences of Sn and Te isotopes having the same neutron number [11]. Strikingly, this predicted variation of $E(2_1^+)$ with n by SMPN, is similar to the variation of experimental E(2_1^+) of ee $^{18-22}$O and $^{42-48}$Ca with ν, clearly showing that the N = 84-88 spectra exhibit the effect of gradual filling of $\nu(2f_{7/2})$ orbital which finally culminates in a new shell closure at^{140}Sn (N= 90) (Fig.1). The realistic CWG does not show these features. Since some important tbmes of the SMPN are tuned to the characteristic (exotic n-richness) properties of the local region i.e. 2-valence particle nuclei above the core, the SMPN, as expected, predicted very well the structure properties of all nuclei in the range as mentioned. Fig.1 shows that if the prediction by SMPN is accepted as correct then there is a universality of behaviour of the n-rich isotopes above different core nuclei.

A. The effective single-particle energies

It is expected that the shell closure at ^{140}Sn should also be manifested in the effective single particle energy (ESPE)[12] diagram as an increased energy gap between two relevant single particle orbitals at N=90. The calculation of ESPEs has been discussed at length in Ref. [12]. The ESPE is defined as the bare single-particle energy (SPE) added to the monopole part contributed by the diagonal TBMEs. The bare SPE is originated from the interaction of a valence nucleon with the nucleons of the doubly closed core. The monopole interaction contribution is the (2J + 1) weighted average of the diagonal TBME, which arises from the interaction of a valence nucleon with the other valence nucleons. The monopole component of an interaction, V, is given by [13].

$$V_{j,j'}^{T} = \frac{\sum_J (2J+1) < j,j'|V|j,j' >_{JT}}{\sum_J (2J+1)} \quad (1)$$

Here the diagonal matrix element of a state where two nucleons in orbits j and $j'$are coupled to an angular momentum J and an isospin T satisfying antisymmetrisation is given by $< j,j'|V|j,j' >_{JT}$. In the present situation, for Sn isotopes, as some neutrons are added to another orbit j', the single-particle energy of the orbit j is changed. The shift of the single-particle energy (*spe*) of j is given by

$$\Delta\epsilon_n(j) = V_{j,j'}^{T=1}\ n_n(j') \quad (2)$$

where $n_n(j')$ is (the expectation value of) the number of neutrons in the orbit j'. After inclusion of this monopole effect, the single particle energy, is called the effective single-particle energy (ESPE). If only protons are added to the j' orbit, the shift in neutron *spe* is given by

$$\Delta\epsilon_n(j) = \frac{1}{2}(V_{j,j'}^{T=1} + V_{j,j'}^{T=0})\ n_p(j') \quad (3)$$

1. The neutron ESPEs for Sn isotopes

It has been discussed in Ref.[9] and shown in Fig.2, how the ESPEs for the configurations $\nu(2f_{7/2})^n$ in $^{132-140}Sn$ with valence neutron number n varying from 0 to 8 evolve. The bare energy difference between two lowest neutron orbits, $\nu(2f_{7/2})$ and $\nu(3p_{3/2})$ is 854 keV for both the Hamiltonians. However, this difference between the corresponding ESPEs increases to 2.246 MeV at N = 90 with SMPN, which is sufficient to make ^{140}Sn a doubly magic nucleus. For CWG this gap does not show any increase but instead decreases slightly to 826 keV.

2. The ESPEs for Z ≥54

Experimental evidences [14] indicate that N = 90 is suitable for onset of deformation for nuclei above Sn (like Xe, Ba, etc.). However, the N = 90 isotope of Sn nucleus, shows a distinct shell closure as predicted by the SMPN results. It is known that the presence of valence protons above the inert core is essential [15] for onset of collectivity. So how the shell closure for N=90 with Z=50 is washed out by the involvement of more protons also needs to be explained. The neutron ESPSs for N=90 as function of valence proton number have been shown in Fig.3. But it is found that introduction of additional protons above Z=50 till Z=58, does not produce any notable change in the energy differences between lowest orbitals. So we investigated the behaviour of the proton ESPEs with increasing number of valence neutrons and protons (Fig.4). In the first panel, Fig.4, the protons ESPEs for Z=51 has been shown for increasing neutron numbers. They do not show any variation. But, it has been discusses and shown in Ref. [9] that if the

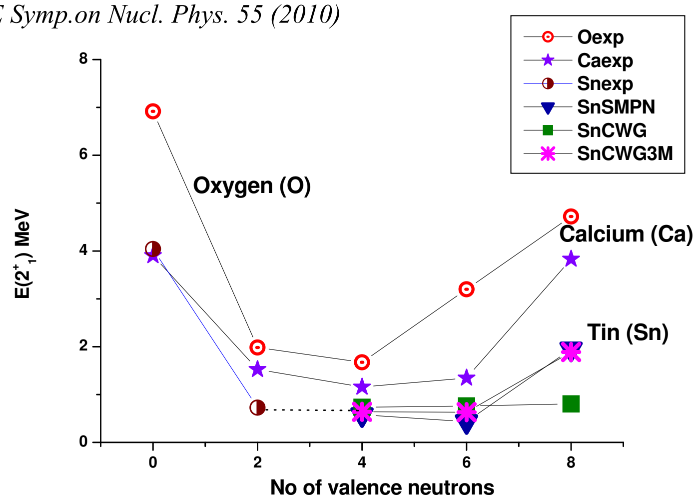

FIG. 1: Results for even Sn isotopes with different interactions compared with neutron rich isotopes of oxygen and calcium.

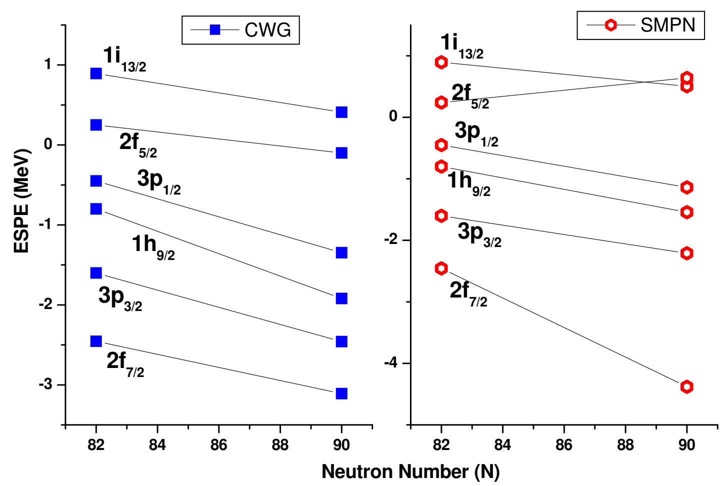

FIG. 2: Variation of neutron ESPE with increasing valence neutron number for SMPN and CWG interactions.

ESPEs for proton orbitals for N = 90 with $\nu(2f_{7/2})^8$ are plotted, substantial reduction of the $\pi(1g_{7/2})$ and $\pi(2d_{5/2})$ energy gap is observed with increasing Z (Fig.4, second panel). This kind of reduction of ESPE gaps might favour enormous configuration mixing thereby leading to onset of collectivity for $Z \geq 54$.

B. The spin-tensor decomposition

To understand the origin of this new shell closure, the physical aspects of both the residual interactions are extracted by a spin-tensor decomposition [16] of the TBMEs. Spin-tensor decomposition of the two-particle interaction is a method which allows separation of the central, vector, and tensor parts of the effective interaction, thus allowing to study their roles in nuclear shell structure and shell evolution. The nomenclature has been adopted from Ref. [17]. They are central, antisymmetric spin-orbit (ALS), spin-orbit (LS), and tensor. We have discussed this decomposition in detail in Ref. [9]. It has been found that for SMPN, the central and ALS part for $2f_{7/2}-2f_{7/2}$ TBMEs account for a majority of the downward shift of the ESPE of $2f_{7/2}$ with

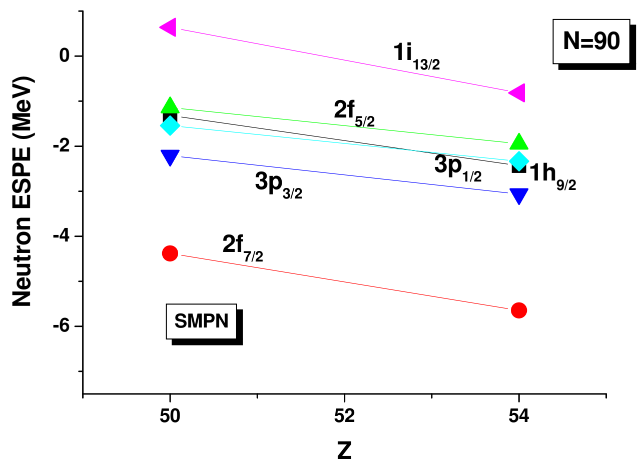

FIG. 3: Variation of neutron ESPE with increasing valence proton number for SMPN interaction.

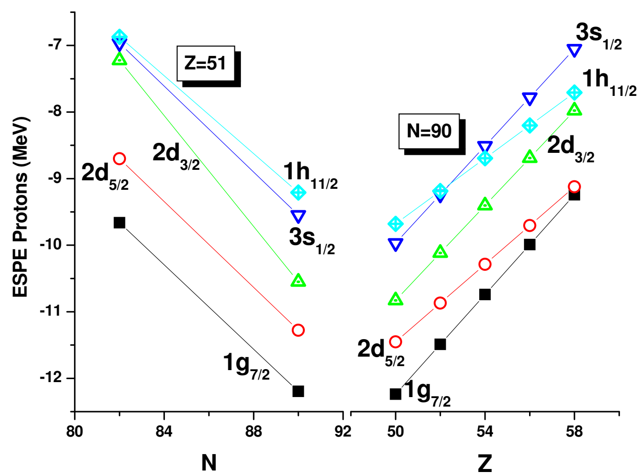

FIG. 4: Variation of proton ESPE with increasing valence neutron and proton numbers for SMPN interaction.

increasing valence neutron number (N) [Fig.2, SMPN results]. The TBMEs involving $3p_{3/2}$ have dominant contribution from the central part. As the central parts of $2f_{7/2}$ and $3p_{3/2}$ vary with similar slopes for increasing neutron numbers, the variation in ALS part is primarily responsible for the observed shell gap at N = 90.

The ALS component in the TBMEs corresponds to the LS-coupled matrix elements which are nondiagonal in S (spin). These terms therefore do not conserve total spin of the matrix elements [18]. There have been several discussions on the origin of strong ALS components usually prevalent in empirical effective interactions. There have been suggestions [9] that it indicates the important contributions from higher-order renormalization or many body forces to the effective interactions.

C. The three-body effects

It has been discussed for a long time that shell model calculations using two-body realistic interactions derived from the free nucleon-nucleon force usually fail to reproduce some shell closures [19]. It has been shown that the three-body forces have to be taken into account to reproduce the shell gaps for neutron rich oxygen and calcium isotopes. Zuker had shown earlier [20] that a very simple three-body monopole term can solve practically all the spectroscopic problems in the p, sd, and fp shells those were earlier assumed to need drastic revisions of the realistic two-body potentials. Recently, Otsuka et al. [21] have shown that three-body forces are necessary to explain drip line for oxygen isotopes.

Therefore, we have incorporated [9] a simple three-body monopole term in CWG as prescribed by Zuker in Ref. [20]. We have incorporated corrections in certain TBMES similar to those in KB3 interaction in pf shell. To compensate for the different mass regions of application of these two interactions, *viz.*, for KB3 (A $>$ 40) and the present case (A $>$ 132), the effect of mass scaling has been included. The scaling is given by $(40/132)^{(1/3)}$ factor. The correction factor will be effective for nuclei for which the valence neutron number N $\geq$ 3. A shell gap for N=90 now appears with CWG3M (CWG+3- body monopole correction term) which is very close to that with SMPN (Fig.1). The SMPN being an empirical interaction, contains in its tbmes, the 3-body effects. The emperical USD predict similarly the shell closure for all the observed shell closed isotopes of oxygen and spin-tensor decomposition of this interaction shows similar large ALS component variation [11]. This provides additional confidence regarding the prediction of shell closure at ^{140}Sn by SMPN interaction. Estimates of the 3-body effects for this mass region by relating the $H^{(p)}$, the p-body matrix elements, to the total many-body matrix element of H = $\sum H^{(p)}$ (experimental) using Racah-algebra as well as empirical fits are in progress.

D. The new mass data and SMPN results

A high-precision direct Penning trap mass measurement has been reported [22] for the isotopes $^{127,131-134}Sn$, which has led to the conclusion that the new shell gap value for the short-lived ^{132}Sn is larger than that of the doubly magic stable ^{48}Ca. Very recently, the measurement of spectroscopic factors [23] indicating purity of the measured single-particle states, clearly illustrates the magic nature of ^{132}Sn. The effect of these newly measured masses have been included in the spe of SMPN and the binding energies of several isotopes in this mass region have been calculated and shown in Table II. The more precise values [22] of masses for $^{132,133}Sn$ compared to Ref. [24] have been used to recalculate the neutron spes. SMPNEW interaction calculation includes these new spes. The Table II clearly shows that the binding energy prediction using SMPNEW is definitely better than that predicted by SMPN. In the column marked NATURESPE, apart from using the precise masses of $^{132,133}Sn$, the newly proposed value for $3p_{1/2}$ spe from [23] has also been included. Although the binding energies do not show any further improvement compared to SMPNEW, the effect on the excited states need to be investigated.

4. Conclusions

The comparison with the systematics of 2_1^+ energies of other n-rich domains and the spin-tensor decomposition of the two interactions establish that new shell closure at ^{140}Sn is definitely a strong possibility. The ALS term, which incorporates in it the contributions of many-body forces contribute substantially in the ESPE of $2f_{7/2}$ of the empirical interaction SMPN. It is found to be responsible for the gap observed in SMPN results. The CWG indicates a weak shell closure at ^{140}Sn. The new CWG3M, which includes simple three-body monopole term with CWG, predicts a shell gap at N = 90 for Sn isotopes as well as decreasing 2_1^+ energies for $^{136,138}Sn$, similar to that from SMPN. This also indicates that the three-body effect plays an important role for shell evolution in neutron-rich Sn isotopes above ^{132}Sn, as also observed in *sd* and *fp* shells. The anomalously depressed 2_1^+ states in Sn isotopes having N = 84-88 and the new shell closure for N = 90 might have interesting consequences for the r-process nucleosyn-

TABLE I: The relevant tbmes of CW5082 [3], CWG [6] and SMPN [2] have been compared. The valence orbitals have been enumerated according to the following convention: PROTONS : $1g_{7/2}(1)$, $2d_{5/2}(2)$, $2d_{3/2}(3)$, $3s_{1/2}(4)$, $1h_{11/2}(5)$; NEUTRONS: $1h_{9/2}(6)$, $2f_{7/2}(7)$, $2f_{5/2}(8)$, $3p_{3/2}(9)$, $3p_{1/2}(10)$, $1i_{13/2}(11)$

Indices				J	T	Hamiltonians		
						CW5082	CWG	SMPN
n-p tbmes								
1	7	1	7	0	0	-0.6778	-0.7922	-0.7355
1	7	1	7	1	0	-0.336	-0.5390	-0.743
1	7	1	7	2	0	-0.29336119	-0.3739	-0.4067
1	7	1	7	3	0	-0.30348513	-0.2385	-0.354
1	7	1	7	4	0	-0.09123172	-0.1290	-0.227
1	7	1	7	7	0	-0.42997605	-0.5440	-0.535
1	6	1	6	8	0	-0.68749624	-0.9491	-1.037
1	11	1	11	10	0	-0.61499959	-0.7795	-0.764
5	7	5	7	9	0	-0.57939130	-0.5890	-1.058
5	11	5	11	10	0	-0.08622793	-0.2972	-1.942
5	11	5	11	11	0	-0.12218534	-0.0354	-1.611
5	11	5	11	12	0	-0.75382543	-1.1495	-1.519
p-p tbmes								
1	1	1	1	0	1	-0.66390	-0.9972	-0.56100
1	1	1	1	2	1	0.16450	-0.2838	0.2050
1	1	1	1	4	1	0.38240	-0.0257	0.5682
1	1	1	1	6	1	0.43640	0.1008	0.5120
n-n tbmes: pairing terms								
6	6	6	6	0	1	-0.61197406	-1.2944	-0.293747544
7	7	7	7	0	1	-0.48571584	-0.6718	-0.233143598
8	8	8	8	0	1	-0.27602252	-0.4515	-0.132490814
9	9	9	9	0	1	-0.37947276	-0.5884	-0.182146922
10	10	10	10	0	1	-0.10030835	-0.1606	-0.0481480062
11	11	11	11	0	1	-0.70925683	-0.9751	-0.340443283
n-n tbmes								
7	7	7	7	2	1	-0.31314358	-0.2983	-0.453
7	7	7	7	4	1	-0.05632162	-0.0909	-0.29
7	7	7	7	6	1	0.05457612	0.0011	-0.162
6	7	6	7	8	1	-0.25914928	-0.3974	-0.495

thesis.

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TABLE II: The comparison of calculated and theoretical binding energy values using CW5082, SMPN and with new spes for SMPN. The experimental values [24] are also shown. Experimental errors are shown within parentheses.

Nucleus	Binding Energy (MeV)				
	Expt.	Theoretical			
[24]		CW5082 [4]	SMPN [2]	SMPNEW [22]	NATSPE [23]
^{134}Sn	6.365(104)	6.705	6.363	6.195	6.211
		5.915(150)^{a}			
^{136}Te	28.564(55)	28.86	28.907	28.739	28.757
^{136}Sn	12.208(501)	13.162	13.041	12.705	12.728
^{136}Sb	19.516(301)	19.759	20.306	20.055	20.067
^{135}Sb	16.565(113)	17.017	16.989	16.821	16.836
^{137}I	37.934(37)	38.011	38.131	37.963	37.981
^{137}Te	31.775(122)	31.762	32.345	32.093	32.109
^{135}Sn	8.437(401)	8.926	9.053	8.801	8.814

^{a} The binding energy value from Ref. [22].