Complexity of energy levels of light nuclei

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Introduction

Complexity of energy spectra of nuclei has remained an important area of research. The subject of random matrix theory (RMT) emerged out of an attempt by Wigner to answer questions related to quantification of complexity. However, most of the work in RMT requires one to possess a large number of levels in a single symmetry class [1]. This restriction is important as it eventually leads us to identify a given sequence of energy levels to a universality class. However, it is very difficult to possess long sequences for nuclei, particularly for lighter nuclei. In this article, we present a way to characterize the complexity of level sequences of lighter nuclei. We extend the ideas of Kolmogorov to realize our goal. We have applied the concept used here to distinguish one quasi-particle and three quai-particle states in the past [2].

Given a level sequence, $\{E_n\}$ for a nucleus of mass number A. The cumulative level density is $N(E) = \sum_i g_i \Theta(E - E_i)$ where g_i give the degeneracy of E_i. We define the Kolmogorov parameter for a sequence of length n as:

$$\lambda_n = \sup_E \frac{N(E) - N'(E)}{\sqrt{n}} \qquad (1)$$

where $N'(E)$ is a theoretical counting function. In large nuclei where due to large number of energy levels, expressions for the average level density known from the works of Thomas and Fermi, or, from the work of Weyl, can be used. However, due to a small number of levels in the nuclei we are considering, $N'(E)$ may simply be quantity E/S where S is the mean-level spacing among the nearest neighbours. For a given sequence of energy levels of length, n, we then divide in intervals of certain length, L. In each of these intervals, we calculate the value of λ and extract the supremum of the calculated values. To infer the degree of stochasticity of the given level sequence, we need the limiting distribution of the parameter, λ. It is proved by Kolmogorov that the distribution is [3]

$$\Phi(\lambda) = \sum_{k=-\infty}^{\infty} (-1)^k e^{-2k^2\lambda^2}. \qquad (2)$$

For the stochasticity parameter, $\lambda = \Lambda$ for a given sequence, the measure of stochasticity is $\Phi(\Lambda)$.

Kolmogorov stochasticity vs random matrix theory

In this article, we consider the energy levels of two nuclei belonging to the sd-shell, ${}^{17}_{9}F$ and ${}^{28}_{14}Si$. These levels belong to the latest compilation based on experimental results [4]. We would like to recall that the work of Kolmogorov has been used widly with great success, first by Kolmogorov himself in defending Mendel's law of inheritance, and later, by Arnold in classifying number-theoretic sequences. By taking (modular) arithmetic and geometric sequences with just 15 terms, Arnold showed that the geometric sequences are about hundred times more complex than arithmetic ones [5]. As described above, random matrix theory is employed analyse spectral fluctuations. However, such an analysis requires a large sequence containing hundreds of energy levels. To facilitate an analysis of RMT, shell model calulations are done to calculate enough energy levels. In this way, for ${}^{28}_{14}Si$, a collection of states with $J^\pi = 2^+$,

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$T = 0$ show that the nearest-neighbour level spacing statistics as well as the spectral rigidity follow closely the Wigner-Dyson statistics for the Gaussian Orthogonal Ensemble (GOE) [6]. However, we could never be sure if the $^{28}_{14}Si$ nucleus does belong to GOE because only 17 energy levels are known with confidence. There also exists an important result for the nuclei over all the mass regions where it is expected that there will be level repulsion even in nuclei of mass numbers less than 50 [7]. However, these calculations are done by considering a nuclear data ensemble where energy levels from many neclei are taken together belonging to the same symmetry class after suitable scaling. In this article, it is the *first* time that a distinctive result on stochasticity of a single nucleus is calculated.

Results and interpretation

Using the method described above, we obtain the following results for the two nuclei:

$$\begin{aligned} ^{17}_{9}F &: \quad \Lambda \approx 0.427; \quad \Phi(0.427) = 0.007, \\ ^{28}_{14}Si &: \quad \Lambda \approx 0.931; \quad \Phi(0.931) = 0.643. \end{aligned} \tag{3}$$

Among the two, Si nucleus has a very (about 90 times more) complex spectrum than F nucleus. The result for Si nucleus stands in happy agreement with the results of RMT. We would thus like to explain why the F nucleus should be less chaotic than Si nucleus.

In order to understand this finding we have to look at the structure of these two systems in terms of single-particle model. $^{17}_{9}F$ is the first member of the sd-shell where the odd proton fills the single particle state in the sd shell. On the other hand, $^{28}_{14}Si$ is the last member of this shell where this shell is completely filled. In the case of $^{17}_{9}F$, the last proton moves only under the influence of the core (^{16}O). The effective potential will be spherically symmetric and the motion of the lone proton is integrable. In the case of $^{28}_{14}Si$, all the nucleons will fill all the m_z-states in the sd shell. Each nucleon will have another nucleon in the m_z-state, scattering each other under pairing interaction. Thus appears a repulsive core experienced by each nucleon along with an asymmetric effective interaction. The single-particle motion will be chaotic. Thus, we have demonstrated that it is this chaotic motion of single particles which leads to spectral fluctuations dictated by GOE of random matrices.

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