Hanging nature of superdeformed bands: A reality

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Introduction

Taking a lead from our experiences of SCQ techniques and the classical analysis of Bohr and Mottelson [1], we present a dynamical scenario of the cranking model for explaining the hanging nature of SD bands. The Hamiltonian of the cranking model of the nucleus:

$$H = \frac{p_x^2 + p_y^2 + p_z^2}{2m} + \frac{1}{2}m\omega^2(x^2 + y^2) + \frac{1}{2}m\omega_z^2 z^2 - \Omega\, l_x \qquad (1)$$

is invariant under the joint action of parity and time reversal (PT) symmetry using an appropriate choice of parameter ($\Omega \rightarrow -\Omega$ under T) [2] . Since the SD bands generally involve a large angular momenta, therefore the term $-\Omega\, s_x$ can be omitted from the Hamiltonian.

It is well known from the quantum mechanics that two commutating linear operators A and B have common eigenfunctions. However, if one of the operators is anti-linear (i.e. T), the simultaneity of eigenfunctions holds only if their eigenvalues are real. Recent investigations [2, 3] have revealed that the cranking Hamiltonian (1) loses its Hermiticity within the region $\omega_z < \Omega < \omega$ and results in spontaneous breakdown of PT symmetry.

The Hamilton equations of motion in the variables y, z, p_y and p_z are written as:

$$\frac{dr_k}{dt} = \frac{\partial H}{\partial p_k}$$

$$\frac{dp_k}{dt} = -\frac{\partial H}{\partial r_k}$$

where k=2,3. These equations reduce to the second order differential equations as:

$$\frac{d^2y}{dt^2} + (\Omega^2 + \omega^2)y - \frac{2\Omega}{m}p_z = 0$$

$$\frac{d^2z}{dt^2} + (\Omega^2 + \omega_z^2)z + \frac{2\Omega}{m}p_y = 0$$

$$\frac{d^2p_y}{dt^2} + (\Omega^2 + \omega^2)p_y + m(\omega^2 + \omega_z^2)\Omega z = 0$$

$$\frac{d^2p_z}{dt^2} + (\Omega^2 + {\omega_z}^2)p_z - m(\omega^2 + \omega_z^2)\Omega y = 0$$

In the vector notation, these expressions become

$$\frac{d^2\boldsymbol{\eta}}{dt^2} + M\boldsymbol{\eta} = 0$$

where M is a constant matrix. The solution of the above equation is given by:

$$\boldsymbol{\eta}(t) = \boldsymbol{\eta}(0)\mathrm{Cos}\sqrt{M}\,t + \frac{\dot{\boldsymbol{\eta}}(0)}{\sqrt{M}}\mathrm{Sin}\sqrt{M}t \qquad (2)$$

with initial conditions $\boldsymbol{\eta}(0)$ and $\dot{\boldsymbol{\eta}}(0)$.

Four eigenvalues of the matrix M are $\pm\lambda_\pm$, where $\lambda_\pm$ is given by

$$\lambda_\pm = \sqrt{\Omega^2 + \omega_+^2 \pm \sqrt{4\Omega^2\omega_+^2 + \omega_-^4}}$$

$$\text{with} \quad \omega_\pm^2 = \frac{\omega^2 \pm {\omega_z}^2}{2} \qquad (3)$$

Results and discussion

Most of the SD bands are normally seen at a large prolate deformation corresponding to the oscillator frequency ratio $\omega:\omega_z$ equal to 1.65:1. This ratio can nearly be expressed as 3:2 and the corresponding quadrupole deformation parameter

$Q = (\kappa\beta\sqrt{\frac{5}{4\pi}}\frac{3}{4j(j+1)}$; κ, β , and j being, respectively, the radial matrix element, deformation of the nucleus and single particle angular momentum) comes out to be $\frac{14}{j^2}$.

A complete discussion of the dynamics of cranking model [1, 4] reveals the existence of twin stable fixed points $\tilde{c}\pm$ phase separated from the usual fixed points corresponding to the aligned ã and anti-aligned $\tilde{b}$ configurations. The twin fixed points $\tilde{c}\pm$ lie in the regime of rotational frequency $\Omega < 2\,Q\,j$ and support the SD band structure [5]. Taking j equal to 7ђ, an upper limit for the cranking frequency Ω is fixed ~ 3.5 for the SD bands. In that discussion, it has also been mentioned that the ND states with $Q \sim \frac{7}{j^2}$ correspond to the aligned configuration ã, which fixes a lower limit~ 1.5 for Ω.

Using the above values ω and ω_z in equation (3), the eigenvalues are obtained by varying the cranking frequency within the limits $1.5 \leq \Omega \leq 3.5$ and are shown in Fig. 1.

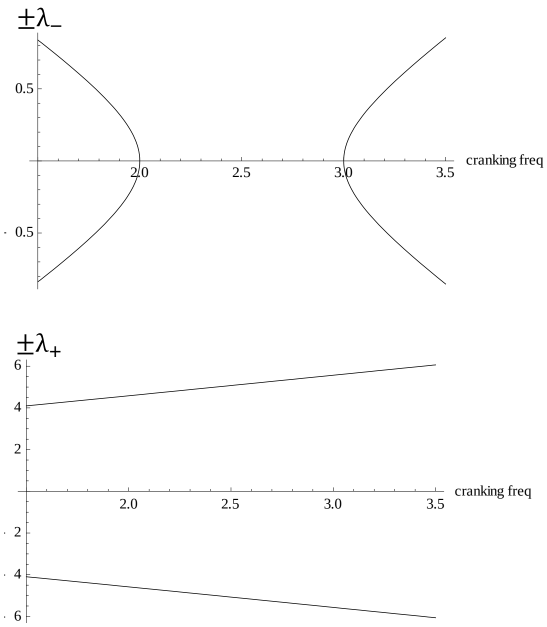

Fig.1 Eigenvalue spectrum $\pm\lambda_-$ and $\pm \lambda_+$vs. the cranking frequency (Ω).

It is evident from this plot that the values of $\pm\lambda_+$ are quite large and vary linearly with Ω .If we vary Ω by a small amount, the variation of $\pm\lambda_+$produces an enormous change in phase and cos and the sin arguments of our solutions (equation (2)) oscillates exceedingly rapidly between plus and minus values. So, the contribution to path is either zero or very small.

Whereas, the $\pm\lambda_-$ are quite reasonable for arguments of cos and sin terms and both these terms remain positive for all the real values of $\pm\lambda_-$. In this case, all the contributions to the path are in phase and do not cancel out. So, the trajectories which mainly arise from $\pm\lambda_-$ contribute significantly to the amplitude.

When the $\Omega < \omega_z$ (i.e. = 1.5), all the eigenvalues (Fig.1) are real and the corresponding solutions (equation (2)) are periodic in character which are invariant under the time-reversal (T) symmetry. Also, the vectors $\boldsymbol{\eta}(0)$, $\dot{\boldsymbol{\eta}}(0)$ and $\boldsymbol{\eta}(t)$ change sign under parity. Therefore, these solutions are invariant under the parity operator and hence PT symmetric.

For $\omega_z<\Omega<\omega$, the eigenvalue λ_- becomes imaginary giving birth to the sinh and cosh terms in (2) which spoil the periodicity of our solutions. Such aperiodic solutions violate the time reversal symmetry (T). The diverging nature of these solutions is associated with the onset of instability. But parity still remains invariant due to vector character of the physical quantity. Thus a conditional regime of rotational frequency $\omega_z<\Omega<\omega$ ensures the breakdown of PT symmetry.

Finally, the eigenvalues $\pm\lambda_-$ become real with an increase of Ω just beyond ω (Fig.1) and restore the periodic nature of solutions (equation (2)). Hence, the PT symmetry is restored in the $\Omega > \omega$ regime.

Conclusion

Parallel to quantum mechanics, we have established the classical guidelines for conservation of PT symmetry. It is noticed that the solutions are no longer periodic in conditional regime (i.e. $\omega_z<\Omega<\omega$). The chaotic region divides our phase space into slow (normal deformed) and fast (superdeformed) rotation. So PT symmetry is an essential ingredient for the inter and intra-band transitions. The breakdown of PT symmetry forbids the linking transitions between the two rotational regimes. This clearly signifies the hanging nature of SD bands is a reality.

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