Anomaly of kinetic MoI expression for shape transitional nuclei

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The deformed, medium mass, even Z-even N nuclei exhibit regular rotational bands based on ground states, and K-bands based on excited states. Using heavy ion (HI) projectiles, the bands are excited to high spin, and regular γ-ray peaks are obtained in the γ-ray spectra. One usually studies the g.s. band structure in terms of the kinetic moment of inertia $J^{(1)}$ using the formula[1]

$$J^{(1)}= (2I-1)/[E(I)-E(I-2)] = (2I-1)/E_\gamma , \quad (1)$$

derived from the rotor formula. Also one studies the changes in the band structure by deducing the dynamic moment of inertia $J^{(2)}$, expressed as [1]

$$J^{(2)}= 4/[E(\gamma_1) – E(\gamma_2)] \quad (2)$$

Incidentally, the same expression is used for all nuclei, rotational or shape transitional and even for near spherical vibrators. This leads to the derived conclusions not according to physical reality. In the present work we discuss this anomaly and illustrate its pitfalls. Using the Rotation-Vibration energy expression
:

$$E(I)=aI(I+1)+bI \quad (3)$$

for spins 2, 4, 6, … or I, one gets the linear relation of the energy ratio $R_{I/2}$ versus $R_{4/2}$, [2]

$$R_{I/2}=(R_{4/2})\, I(I-2)/8 - I(I-4)/4. \quad (4)$$

Thereby, one gets all $R_{I/2}$, the calculated level energies E_I^{linear}, E_γ^{linear} and moments of inertia $J^{(1)}_{linear}$ and $J^{(2)}_{linear}$ [3] . In Table 1 we illustrate it for $R_{4/2}$=3.0 and 2.25 and show that the ground state shape itself determines the slope of the kinetic MoI curves for all nuclei.

The numbers in (Table 1) are given in units of $E(2_1)$. Here we note that w*ith increasing spherical content and the decreasing $R_{4/2}$ the slope of the kinetic MoI versus spin increases.* For $R_{4/2}$=3.0 and 2.25, the linear MoI rises from 3.0 to 4 and 12 respectively at $I=18^+$. The difference between $J^{(1)}_{expt}$ and $J^{(1)}_{linear}$ represents the effect of the rotation-vibration interaction, which increases with spin. Only these differences represent the structural changes with spin. The slope of the linear MoI curve is a measure of the sphericity of the nuclear core in the ground state itself and it overlaps the slope of $J^{(1)}_{expt}$ at the lower spin (Fig.1) [3].

Table 1. The values of $R_{I/2}$ or E(I), relative to E(2), given by Eq. 4.

$R_{4/2}$	Spin I	2	4	6	8	10	12	14	16	18
3	E(I)	1	3	6	10	15	21	28	36	45
	E_γ	1	2	3	4	5	6	7	8	9
	$E_\gamma^{I+2}-E_\gamma^{I}$	1	1	1	1	1	1	1	1	
	$J_I^{(1)}$	3	3.5	11/3	15/4	19/5	23/6	27/7	31/8	35/9
	$J_I^{(2)}$	4	4	4	4	4	4	4	4	4
2.25	E(I)	1	2.25	3.75	5.5	7.5	9.75	12.25	15.0	18
	E_γ	1	1.25	1.50	1.75	2	2.25	2.5	2.75	3
	$E_\gamma^{I+2}-E_\gamma^{I}$	0.25 constant								
	$J_I^{(1)}$	3	5.6	7.3	8.6	9.5	10.2	10.8	11.0	11.7.
	$J_I^{(2)}$	16	constant							

For the linear relation assumed in Eq. (3), the dynamic moment of inertia $J_I^{(2)}$ stays constant for all spins I, but this constant value is dependent on $R_{4/2}$ and increases with decreasing $R_{4/2}$. For $R_{4/2}$=3.0 and 2.25 the dynamic MoI assumes a constant value of 4 and 16 respectively.

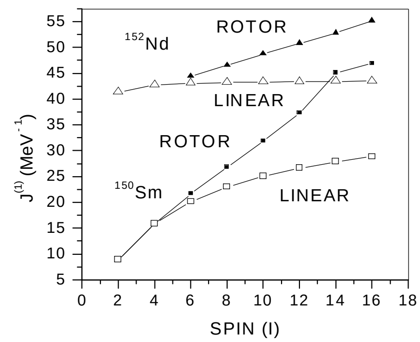

Fig. 1. The plot of kinetic moment of inertia versus spin I for ^{152}Nd and ^{150}Sm.

For ^{150}Sm, we illustrate (Table 2) the prediction from Eq. 4 and demonstrate that in ^{150}Sm ($R_{4/2}$=2.32), the kinetic MoI increases from 9.0 to 45.1 at 14^+, i.e. 5-fold increase, out of which the 3-fold increase is on account of core sphericity!

Table 2. The kinematic moment of inertia, using E_γ (expt.) and from $R_{I/2}$.(MeV^{-1}).

Isotope R4/2	2^+	4^+	6^+	8^+	10^+	12^+	14^+	16^+
^{150}Sm (2.316)								
E(keV)	333.9	773.4	1278.9	1837.0	2432.0	3048.0	3646.0	4306.0
$J^{(1)}$.	9.0	15.93	21.76	26.88	31.93	37.34	45.15	46.97
Lin MoI	9.0	15.93	20.2	23.1	25.1	26.7	27.9	28.9
Diff.	0	0	1.6	3.8	6.8	10.7	17.2	18.1

This partly explains the observed increasing slopes of $J^{(1)}_{expt}$ against spin I or ω (or ω^2). The difference in $J^{(1)}_{expt}$ and $J^{(1)}_{linear}$ is a better measure of the variation of nuclear structure. For^{152}Nd, with $R_{4/2}$=3.0, the difference between the kinetic moment of inertia obtained from Eq. (1) and the linear MoI from equation 4 is much less. Full implications of this are discussed in a longer Paper [3].

Conclusion

The noteworthy inference from present study is that the degree of deformation in the ground state itself determines the slope of the kinetic moment of inertia versus spin plots, which can be determined from the energy ratio R(4)=E(4)/E(2). Only the difference between the usual plots based on Eq. (1) and the linear plot based on Eq. (4) are a measure of structural change with spin.

References

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