Double-Λ hypernuclei within a Skyrme-Hartree-Fock approach

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The double-Λ hypernuclei were first observed in the 1960s [1]. Since then great attention has been given to understand the structure of such systems as they provide a unique opportunity to understand the hyperon-hyperon (YY) interaction [2]. They also supplement the information about the hyperon-nucleon (YN) interaction that is mostly extracted from the studies of the single-Λ hypernuclear systems. The knowledge of YN and YY interactions are needed for making extrapolations to understand the properties of both finite as well as bulk strange hadronic matter [4] and the neutron stars [5]. Whereas a large amount of experimental data are available for the single-Λ hypernuclei covering a wide range of the periodic table [3], only a few double-Λ hypernuclei are known with some certainty. Nevertheless, numerous theoretical studies of the double-Λ hypernuclei are available addressing to these few systems. Three-body (mostly $\alpha + \Lambda + \Lambda$) variational methods have been used to study the $^{6}_{\Lambda\Lambda}$He system by several authors [6]. Among other approaches are the Fadeev [7], VMC [8] and stochastic variational six-body [9] calculations.

The Skyrme-Hartree-Fock (SHF) approach is known to be a powerful tool for investigating the gross properties of nonstrange nuclei. It was extended to study the Λ hypernuclei by Rayet [10]. However, reliability of this approach depends of the accurate knowledge of the Skyrme $\Lambda-N$ interaction, which are determined by fitting to the experimental binding energies of known hypernuclei. Whereas most of the SHF studies use $\Lambda-N$ interactions that have been obtained by fitting to only a few systems, we have in a recent study [11], determined this interaction by fitting to the modern data on the binding energies of almost 20 Λ hypernucei. The fit potential with minimum χ^2 has been used to describe successfully the properties of almost all the known (a total of about 93) single-Λ hypernuclei.

In this contribution, we extend our SHF method to the calculations of the double-Λ hypernuclei. This can be done in a straight forward way by defining a $\Lambda\Lambda$ interaction and retaining the NN and ΛN interactions as used before in Ref. [11]. Since data on double-Λ hypernuclei are still scarce, we consider a simplified Skyrme-like $\Lambda\Lambda$ potential as

$$
\begin{aligned}
V_{\Lambda\Lambda} &= \lambda_0\delta(r_1-r_2)+\frac{1}{2}\lambda_1[k'^2\delta(r_1-r_2) \\
&+ \delta(r_1-r_2)k^2]+\lambda_2 k'\delta(r_1-r_2)k \\
&+ \lambda_3\delta(r_1-r_2)\rho_N^{\alpha}\Big(\frac{r_1+r_2}{2}\Big), \qquad (1)
\end{aligned}
$$

where ρ_N is the nucleon density. Other notations are the standard one and are described in [11]. The last term in Eq.(1) corresponds to the three-body $\Lambda\Lambda N$ interaction. The Hartree-Fock equations for baryon single-particle radial wave functions remain the same except for the fact that the single-particle potential U_B and the effective mass m^*_Λ acquire additional terms due to the presence of $V_{\Lambda\Lambda}$. The separation energies B_Λ and $B_{\Lambda\Lambda}$ are evaluated by solving the appropriate Hartree-Fock

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equations and using the relations

$$B_\Lambda({}^{A+1}_{\Lambda}Z) = E_b({}^{A+1}_{\Lambda}Z) - E_b({}^{A}Z), \quad (2)$$

and

$$B_{\Lambda\Lambda}({}^{A+2}_{\Lambda\Lambda}Z) = E_b({}^{A+2}_{\Lambda\Lambda}Z) - E_b({}^{A}Z), \quad (3)$$

where $E_b({}^{A+2}_{\Lambda\Lambda}Z)$, $E_b({}^{A+1}_{\Lambda}Z)$ and $E_b({}^{A}Z)$ denotes the total binding energies of double Λ hypernucleus, single Λ hypernucleus and its non-strange core nucleus, respectively. The main quantity of interest in $\Lambda\Lambda$ hypernuclei is the $\Lambda\Lambda$ bond energy

$$\Delta B_{\Lambda\Lambda} = B_{\Lambda\Lambda} - 2B_\Lambda \quad (4)$$

where $B_{\Lambda\Lambda}$ is the separation energy of the hyperon pair from a ${}^{A+2}_{\Lambda\Lambda}Z$ hypernucleus and B_Λ is the hyperon separation energy from the ${}^{A+1}_{\Lambda}Z$ one.

Main results of our calculations are shown in Table I. We have used the SLy4 and HP120310 interactions for NN and ΛN systems, respectively which are described in Ref. [11]. In $V_{\Lambda\Lambda}$, we have used $\lambda_0 = -437.7(MeV fm^3$, $\lambda_1 = 240.7(MeV fm^5)$, and $\mu = 1.05 fm$. The density dependent term has been dropped in this interaction ($\lambda_3 = 0$). The p-wave interaction amplitudes λ_2 are irrelevant for the ground states which are all s-wave states, so the λ_2 is also set to zero. We note that our model provide a good description of the limited set of the $\Lambda\Lambda$ binding energy data that are available up to now. It is seen that $B_{\Lambda\Lambda}$ increases with mass number of the nucleus, which is also reproduced by our calculations. We also give our predictions for some heavier double-Λ hypernuclei which will be tested in future experiments planned at the JPARC facility in Japan.

TABLE I: $B_{\Lambda\Lambda}$ and $\Delta B_{\Lambda\Lambda}$ of $\Lambda\Lambda$ hyperon in various hypernuclei Quantities given in brackets are the corresponding experimental data.

Hypernuclei	$B_{\Lambda\Lambda}$[MeV]	$\Delta B_{\Lambda\Lambda}$[MeV]
${}^{6}_{\Lambda\Lambda}$He	09.75 (10.06 ± 1.72)	0.79 (0.67 ± 0.17)
${}^{10}_{\Lambda\Lambda}$Be	16.06 (14.94 ± 0.13)	3.87
${}^{11}_{\Lambda\Lambda}$Be	16.97 (16.60 ± 0.80)	5.11 (−1.6 ± 0.8)
${}^{12}_{\Lambda\Lambda}$B	17.75 (18.90 ± 0.90)	6.32 (−1.5 ± 0.9)
${}^{13}_{\Lambda\Lambda}$B	18.35 (23.30 ± 0.70)	7.59 (0.6 ± 0.8)
${}^{16}_{\Lambda\Lambda}$N	19.34	8.96
${}^{18}_{\Lambda\Lambda}$O	19.73	10.11
${}^{42}_{\Lambda\Lambda}$Ca	23.51	16.89
${}^{92}_{\Lambda\Lambda}$Zr	25.70	20.80
${}^{210}_{\Lambda\Lambda}$Pb	27.60	22.80

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