Magnetic moments of the negative parity N^* resonances

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The study of the properties of the $S_{11}(1535)$ N^* resonance, the lowest lying $J^P = \frac{1}{2}^-$ nucleon resonance, provides valuable insight into the nature of QCD in the non-perturbative domain [1]. In particular, the large mass splitting between the nucleon ground state $N(939)$ and its negative parity partner $N^*(1535)$ is connected to the spontaneous breaking of the chiral symmetry. Indeed, if the two-flavor chiral symmetry were exact and preserved by the QCD vacuum, QCD would predict parity doublets degenerate in mass [2, 3].

Experimentally, the magnetic moments of the nucleon resonances can be extracted through bremsstrahlung processes. The measurement of the magnetic moment of $\Delta^{++}(1232)$ has been performed in the process $\pi^+ p \to \gamma\pi^+ p$. New experiment to measure the reaction of $\gamma p \to \gamma\pi^0 p$ has been carried out by the A2/TAPS collaboration at MAMI. This process is expected to avoid the bremsstrahlung contributions from the pions [4]. For the $S_{11}(1535)$ resonance, it is believed that its magnetic moment can be extracted through the process of $\gamma p \to \gamma\eta p$ [5]. Since this resonance strongly couples to the ηN channel, the η meson in the final state can be regarded as a probe of the $S_{11}(1535)$ resonance in the intermediate state.

In the low energy regime, chiral constituent quark model (χCQM) successfully explains the "proton spin crisis" and other related properties [6–8]. In this work, we intend to extend the applicability of the model to study the magnetic moment of the negative-parity low-lying nucleon resonances with orbital angular moment being 1.

1. Magnetic moment in quark model

In the nonrelativistic SU(6) constituent quark model (CQM) [1], the lowest-lying negative-parity nucleon resonances are $|N^2P_{1/2}\rangle$ and $|N^4P_{1/2}\rangle$, where the usual spectroscopic notations $^2P_{1/2}$ and $^4P_{1/2}$ are used to indicate their total quark spin $S = 1/2, 3/2$ $(2S+1 = 2, 4)$, orbital angular momentum $L = 1$ (P-wave), and total angular momentum $J = 1/2$. The wavefunctions of the $|N^2P_{1/2}\rangle$ and $|N^4P_{1/2}\rangle$ states are given explicitly as

$$\begin{aligned}|N^4P_{1/2}\rangle &= \frac{1}{\sqrt{2}}\chi^s\{\psi^\lambda\phi^\lambda + \psi^\rho\phi^\rho\}, \quad (1)\\ |N^2P_{1/2}\rangle &= \frac{1}{2}\{\chi^\lambda\psi^\rho\phi^\rho + \chi^\rho\psi^\lambda\phi^\rho + \\ &\quad \chi^\rho\psi^\rho\phi^\lambda - \chi^\lambda\psi^\lambda\phi^\lambda\}. \quad (2)\end{aligned}$$

where ψ, χ, and ϕ denote the spatial, spin and flavor part of wavefunctions.

The observed lowest-lying negative-parity nucleon resonances are the $S_{11}(1535)$ and $S_{11}(1650)$, obtained as configuration mixtures of the $|N^2P_{1/2}\rangle$ and $|N^4P_{1/2}\rangle$ SU(6) states,

$$|S_{11}(1535)\rangle = |N^2P_{1/2}\rangle\cos\theta - |N^4P_{1/2}\rangle\sin\theta,$$
$$|S_{11}(1650)\rangle = |N^2P_{1/2}\rangle\sin\theta + |N^4P_{1/2}\rangle\cos\theta,$$

where θ denotes the mixing angle. Now the magnetic moments of the $S_{11}(1535)$ and $S_{11}(1650)$ resonances can be expressed in terms of the magnetic moments of the $|N^2P_{1/2}\rangle$ and $|N^4P_{1/2}\rangle$ states and the cross

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terms as

$$\mu(S_{11}(1535)) = \mu(N^2P_{1/2})\cos^2\theta$$
$$-2\,\langle N^2P_{1/2}|\mu_z^S|N^4P_{1/2}\rangle\sin\theta\cos\theta$$
$$+\mu(N^4P_{1/2})\sin^2\theta\,, \quad (3)$$
$$\mu(S_{11}(1650)) = \mu(N^2P_{1/2})\sin^2\theta$$
$$+2\langle N^2P_{1/2}|\mu_z^S|N^4P_{1/2}\rangle\sin\theta\cos\theta$$
$$+\mu(N^4P_{1/2})\cos^2\theta\,. \quad (4)$$

Using the relation $\mu = Q/2m$, we obtain $\mu_u = Q_u/2m_u = 2\,\mu_N$, $\mu_d = Q_d/2m_d = -\mu_N$, and substituting $\theta = -31.7°$ in Eqs. (3) and (4), the magnetic moment of the $S_{11}(1535)$ and $S_{11}(1650)$ states can be calculated and the corresponding results are presented in Table I.

2. Magnetic moment in χCQM

In χCQM, the spin structure and magnetic moment after one interaction can be obtained by substituting for every constituent quark

$$q\uparrow \;\rightarrow\; P_q q\uparrow + |\psi(q\uparrow)|^2,$$
$$q^{(1)} \;\rightarrow\; T_q q^{(1)} + |\psi(q^{(1)})|^2\,, \quad (5)$$

where P_q and T_q are the probabilites of no emission of GBs from a $q\uparrow$ and $q^{(1)}$ quark and $|\psi(q\uparrow)|^2$ and $|\psi(q^{(1)})|^2$ are the probabilities of transforming a $q\uparrow$ and $q^{(1)}$, respectively [7].

The magnetic moment calculations involve the SU(3) symmetry breaking parameters, a, $a\alpha^2$, $a\beta^2$, and $a\zeta^2$ which represent the probabilities of fluctuations to pions, K, η, and $\eta^{'}$, respectively. A best fit of χCQM parameters can be obtained by carrying out a fine grained analysis of the spin and flavor distribution functions leading to

$$a = 0.12\,,\quad \alpha = \beta = 0.45\,,\quad \zeta = -0.15\,.$$

Using the above set of parameters, we have calculated the magnetic moment of the $S_{11}(1535)$ and $S_{11}(1650)$ resonances in CQM and χCQM and the numerical results are presented in Table I.

3. Summary and Conclusions

The χCQM is able to get a fairly good description of $S_{11}(1535)$ and $S_{11}(1650)$ resonances. For the $S_{11}(1535)$ resonance, since it is known to dominate the reaction of $\gamma p \rightarrow \eta p$ in the threshold region, the process $\gamma p \rightarrow \gamma\eta p$ in the soft photon limit is believed a probe to extract the magnetic moment of $S_{11}(1535)$. It is expected that the experiments at Crystal Barrel@ELSA and Crystal Ball@MAMI are promising to measure the magnetic moments of the $S_{11}(1535)$ resonance in near future.

Resonance	CQM	χCQM
$S_{11}^{+}(1535)$	1.89	1.57
$S_{11}^{0}(1535)$	-1.28	-1.06
$S_{11}^{+}(1650)$	0.11	-0.05
$S_{11}^{0}(1650)$	0.95	0.69

TABLE I: Magnetic moments of $S_{11}^{+(0)}(1535)$ and $S_{11}^{+(0)}(1650)$ resonances

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