Embedded random matrix ensemble results for neutrinoless double-beta decay

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Introduction

Fundamental significance of neutrinoless double-beta decay (NDBD) is that its experimental confirmation will tell us about lepton number violation in nature and that neutrino is a Majorana particle. More importantly, NDBD gives a value or a bound on neutrino mass [1] provided the half-lives are known experimentally and the corresponding nuclear transition matrix elements (NTME) are obtained using a reliable nuclear model. At present large number of NDBD experiments are being carried out and the nuclei being considered are ^{48}Ca, ^{76}Ge, ^{82}Se, ^{100}Mo, ^{116}Cd, ^{124}Sn, ^{130}Te, ^{136}Xe, ^{150}Nd and so on [1]. Many nuclear models are used for the calculation of NTME for NDBD with the latest given in [2]. Our purpose is to apply spectral distribution theory and to this end, we establish that the spreading function that enters in the theory is close to a bivariate Gaussian and also derive a formula for bivariate correlation coefficient.

Spectral Distribution Theory

Statistical spectral distribution theory [3] gives a method for calculating transition strengths (squares of transition matrix elements) generated by a transition operator. This theory starts with shell model spectroscopic spaces and the same shell model inputs (single particle energies and effective two-body interactions). Here one constructs smoothed forms (spectral distributions) for various observables ignoring the fluctuations and this is based on random matrix representation of Hamiltonians (also other operators), unitary decompositions of the operators and quantum chaos.

Half-life for 0ν double-beta decay, for the 0_i^+ ground state (gs) of a initial even-even nucleus decay to the 0_f^+ gs of the final even-even nucleus is given by [1]

$$\left[T_{1/2}^{0\nu}\right]^{-1} = G^{0\nu}\left|M^{0\nu}(0^+)\right|^2\left(\frac{\langle m_\nu\rangle}{m_e}\right)^2 \quad (1)$$

where $\langle m_\nu\rangle$ is the effective neutrino mass. The $G^{0\nu}$ is kinematical factor [1]. The $M^{0\nu}$ is the NTME generated by the NDBD transition operator which is a two-body operator and it involves the so-called neutrino potential. The NTME $\left|M^{0\nu}\right|^2$ can be viewed as a transition strength (matrix element connecting a given initial state to a final state by a transition operator) generated by the two-body transition operator $\mathcal{O}(2:0\nu)$. Therefore, spectral distribution theory for transition strength densities [3, 4] can be applied. Transition strength density is defined as the transition strength multiplied by the state densities at the initial and final energies involved.

The nuclear effective Hamiltonian is one plus two-body, $H = \mathbf{h}(1) + \mathbf{V}(2)$ and we assume that the one-body part $\mathbf{h}$ includes the mean-field producing part of the two-body part. Thus, $\mathbf{V}$ is the irreducible two-body part of H. Random matrix theory, based on embedded Gaussian orthogonal ensemble of random matrices generated by random two-body interactions in presence of a mean field [EGOE(1+2)], for the (smoothed) transition strength densities $I_{\mathcal{O}}^H(E_i,E_f) = I(E_f)|\langle E_f \mid \mathcal{O} \mid E_i\rangle|^2 I(E_i)$ allows us to write $I_{\mathcal{O}}$ as a convolution of the corresponding density generated by the mean-field part $\mathbf{h}$ with a spreading bivariate distribution $\rho_{biv:\mathcal{O}:\mathbf{V}}$ due to the interaction $\mathbf{V}$. It is easy to construct $I_{\mathcal{O}}^{\mathbf{h}}$ for the two-body NDBD transition oper-

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ator. Therefore, to complete the theory we need a form for $\rho_{biv:\mathcal{O}:\mathbf{V}}$. Based on the past results for spinless fermion systems [4], it is possible to argue that the spreading function $\rho_{biv:\mathcal{O}:\mathbf{V}}^{(m_p,m_n)_i,(m_p,m_n)_f}$, defined over proton-neutron spaces, should be close to a bivariate Gaussian. We will apply binary correlation approximation method to prove this.

EGOE Binary Correlation Results

In this section, we write $\mathbf{V}$ as H and $\mathcal{O}(2:0\nu)$ as simply $\mathcal{O}$. Firstly, we consider proton-neutron configurations as two-orbit configurations with one orbit (space #1) for protons and other (space #2) for neutrons. Then, H preserves (m_p, m_n) as it is true for nuclei and $H = \sum_{\alpha,\beta,\gamma,\delta}\left[v_H^{\alpha\beta\gamma\delta}(i,j)\right] \quad \alpha_1^\dagger(i)\beta_1(i)\gamma_2^\dagger(j)\delta_2(j)$. Here, $\alpha_1^\dagger(i)$ $[\gamma_2^\dagger(j)]$ is normalized creation operator for i $[j]$ number of particles in orbit #1 [#2]. Similarly, $\beta_1(i)$ and $\delta_2(j)$ are annihilation operators. Note that $i+j=2$ and $v_H^{\alpha\beta\gamma\delta}(i,j)$ are independent Gaussian random variables with zero center and variance independent of $(\alpha,\beta,\gamma,\delta)$, i.e. H is a EGOE in (m_p,m_n) space with $v_H^2(i,j) = \overline{[v_H^{\alpha\beta\gamma\delta}(i,j)]^2}$. Similarly, the general form of the transition operator $\mathcal{O}$, with EGOE representation independent of the H ensemble, is $\mathcal{O} = \sum_{\gamma,\delta} v_{\mathcal{O}}^{\gamma\delta}(2) \;\; \gamma_1^\dagger(2)\delta_2(2)$. Note that $\overline{(v_{\mathcal{O}}^{\gamma\delta})^2} = v_{\mathcal{O}}^2$. Given this, we have extended binary correlation theory given in [5] to two-orbits and derived formulas for the bivariate moments $M_{rs}(m_1,m_2)$ with $r+s \leq 4$ and therefore, for the bivariate cumulants k_{rs}. The formulas are available in detail in [6] and some numerical results are given in Table I. Note that the odd moments will vanish due to EGOE representation and the moments with $r+s=2$ will give variances and correlation coefficient ζ. For a bivariate Gaussian, k_{rs} should be zero for $r+s>2$ and therefore, from Table I, we can infer that the transition strength density generated by transition operator $\mathcal{O}$ will be in general a bivariate Gaussian. Binary correlation theory gives the bivariate correlation coefficient to be 0.57, 0.72, 0.76, 0.77 and 0.83 respectively for the nuclei listed in Table I. As $\zeta \sim 0.6-0.8$ ($\zeta = 0$ for GOE), the transition strength density will be narrow in the (E_i,E_f) plane and therefore, EGOE is relevant for NDBD.

TABLE I: Cumulants k_{rs}, $r+s=4$ for some nuclei appropriate for $0\nu\beta^-\beta^-$ decay operator. The configuration spaces corresponding to N_p or $N_n = 30$, 32, 44 and 58 are r_4g, r_4h, r_5i, and r_6j, respectively with $g = {}^1g_{9/2}$, $h = {}^1h_{11/2}$, $i = {}^1i_{13/2}$, $j = {}^1j_{15/2}$, $r_4 = {}^1g_{7/2}\,{}^2d_{5/2}\,{}^2d_{3/2}\,{}^3s_{1/2}$, $r_5 = {}^1h_{9/2}\,{}^2f_{7/2}\,{}^2f_{5/2}\,{}^3p_{3/2}\,{}^3p_{1/2}$ and $r_6 = {}^1i_{11/2}\,{}^2g_{9/2}\,{}^2g_{7/2}\,{}^3d_{5/2}\,{}^3d_{3/2}\,{}^4s_{1/2}$.

Nuclei	N_p	N_n	k_{40}	k_{04}	k_{13}	k_{31}	k_{22}
${}^{100}_{42}Mo_{58}$	30	32	−0.45	−0.42	−0.24	−0.26	−0.20
${}^{150}_{60}Nd_{90}$	32	44	−0.27	−0.29	−0.22	−0.20	−0.19
${}^{154}_{62}Sm_{92}$	32	44	−0.24	−0.25	−0.19	−0.18	−0.17
${}^{180}_{74}W_{106}$	32	44	−0.19	−0.20	−0.17	−0.15	−0.15
${}^{238}_{92}U_{146}$	44	58	−0.18	−0.18	−0.15	−0.15	−0.13

Now that we have established that the spreading function generated by the NDBD transition operator should be close to a bivariate Gaussian, we are in a position to apply spectral distribution theory to calculate NTME for NDBD. As a first application, calculations are in progress for ^{150}Nd and ^{238}U NDBD.

References

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