Scattering of neutrons from nuclei and the semiclassical optical model

Tapan Mukhopadhyay* and D. N. Basu†
Variable Energy Cyclotron Centre, 1/AF Bidhan Nagar, Kolkata 700 064, India

We derive the total and the differential cross sections with respect to angle for neutron induced reactions from an analytical model having a simple functional form to demonstrate the quantitative agreement with the measured values for energies 5-600 MeV and angles 0-30 degrees respectively. Often, these cross sections are evaluated using phenomenological optical potentials and much effort has gone into defining global sets of parameter values for optical potentials to estimate cross sections as yet unmeasured. The present model is an easy to use analytical parameterization that may be adequate to produce these cross sections needed for various calculations.

The optical potential for nuclear n-N interaction can be written as $-V - iW$ with V and W as positive quantities, and contains no Coulomb interaction. The phase shift δ in a WKB approximation is $[\int K'dr - \int k'dr]$ and the real part of it to a zeroth order approximation for a square well with radius R is $(K-k)R$ where K is the real part of K' and $k' = k$ is real due to absence of potential. The real wave numbers inside and outside the nucleus are, therefore, given by $K^2 = 2m(E+V)/\hbar^2$ and $k^2 = 2mE/\hbar^2$ respectively where E is the incident neutron energy in the center of mass system and m is the reduced mass of the neutron-nucleus system. The average chord length $\bar{R}$ of a neutron passing through a nucleus can be derived as

$$\bar{R} = \frac{\int_0^R 2\sqrt{R^2-x^2}(I2\pi x dx)}{\int_0^R I2\pi x dx} = \frac{4}{3}R \qquad (1)$$

where I is the neutron flux. Since $R \propto A^{\frac{1}{3}}$ ($R \sim r_0 A^{\frac{1}{3}}$), the above arguments imply that β, which is two times the real part of the phase shift, is determined by the real potential V,

$$\beta = 2(K-k)R = \beta_0 A^{\frac{1}{3}}[\sqrt{E+V} - \sqrt{E}] \qquad (2)$$

where $\beta_0 = \frac{2r_0(2m)^{\frac{1}{2}}}{\hbar}$ whose value is approximately 0.6, whereas the attenuation factor α is determined primarily by the imaginary potential W,

$$\alpha = e^{-\bar{R}/\Lambda} = \exp[-\alpha_0 r_0 A^{\frac{1}{3}} W/\sqrt{E+V}] \qquad (3)$$

where $\Lambda = \hbar^2 K/2mW$ is the mean free path of the neutron inside the nucleus, $\alpha_0 = \frac{4(2m)^{\frac{1}{2}}}{3\hbar} = 0.2929$ and r_0 is the nuclear radius parameter. The first term $V_A = V_0 + V_1(1-2Z/A) + V_2/A$ of the real potential $V = V_A + V_E\sqrt{E}$ contains both the isoscalar and the isovector components of the optical potential where Z is the atomic number of the target nucleus, whereas the second term accounts for its energy dependence. The imaginary potential W is taken as $W = W_0 + W_E\sqrt{E+V}$ since the total kinetic energy of the neutron inside the nucleus with attractive potential well of depth V is $E+V$. As magnitude of V decreases and that of W increases with energy, V_E is negative whereas the W_E is positive.

From partial wave analysis, scattering (σ_{sc}) and reaction (σ_r) cross sections are given by

$$\sigma_{sc} = \frac{\pi}{k^2}\Sigma_l\ (2l+1)|1-\eta_l|^2,$$

$$\sigma_r = \frac{\pi}{k^2}\Sigma_l\ (2l+1)[1-|\eta_l|^2]$$

$$\frac{d\sigma_{sc}}{d\Omega} = \frac{1}{4k^2}|\Sigma_l(2l+1)(1-\eta_l)P_l(cos\theta)|^2 \qquad (4)$$

where the quantity $\eta_l = e^{2i\delta_l}$. With the assumption of the semiclassical optical model (or the so called nuclear 'Ramsauer model') that the phase shift δ_l is independent of l and the summation over partial waves is performed upto the sharp cut-off kR_{ch} only, it follows that $\sigma_{sc} = \pi(R_{ch}+\lambda)^2(1+\alpha^2-2\alpha\cos\beta)$,

*E-mail:tkm@vecc.gov.in
†E-mail:dnb@vecc.gov.in

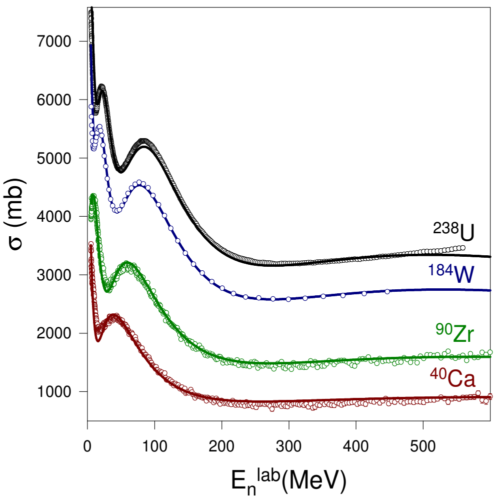

FIG. 1: Plots of σ_{tot} versus incident neutron energies for ^{238}U, ^{184}W, ^{90}Zr and ^{40}Ca target nuclei. The hollow circles represent the experimental data [1–3] and the lines represent the same obtained theoretically with present formulation.

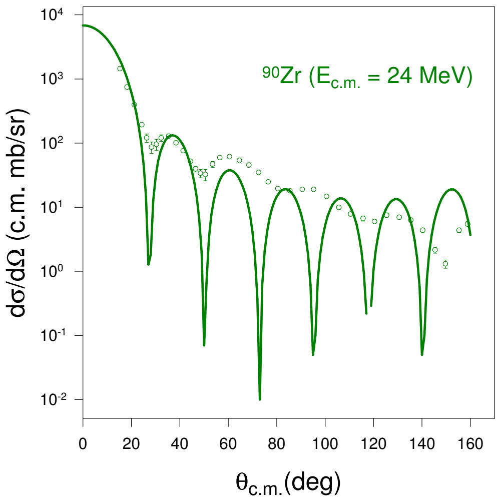

FIG. 2: Plots of theoretical and experimental angular distributions for neutron scattering for ^{90}Zr. The hollow circles represent the experimental data [4] whereas the line represents the same obtained theoretically with present formulation.

$\sigma_r = \pi(R_{ch}+\lambda)^2(1-\alpha^2)$ where $\lambda = 1/k$, R_{ch} is the channel radius beyond which partial waves do not contribute, $\beta = 2\text{Re}\delta_l = 2\text{Re}\delta$, $\alpha = e^{-2\text{Im}\delta_l} = e^{-2\text{Im}\delta}$ and summing over l from 0 to kR_{ch} yielded $\Sigma_{l=0}^{kR_{ch}}\ (2l+1) = (kR_{ch}+1)^2$.

$$\sigma_{tot} = \sigma_{sc} + \sigma_r = 2\pi(R_{ch}+\lambda)^2(1-\alpha\cos\beta)$$

$$\frac{d\sigma_{sc}}{d\Omega} = \frac{\lambda^2}{4}(1+\alpha^2-2\alpha\cos\beta)[\sum_{l=0}^{kR_{ch}}(2l+1)P_l]^2 \quad (5)$$

The present model can be fitted to the experimental neutron total cross section σ_{tot}. The radius of the nuclear potential is given by $R = r_0A^{\frac{1}{3}}$ whereas the channel radius can be parametrized as $R_{ch} = r_0A^{\frac{1}{3}} + r_A\sqrt{E} + r_2$ with $r_A = r_{10}\ \ln A + r_{11}/\ln A$. The fits yield $r_{10} = -22.98\times10^{-3}$, $r_{11} = 10.27\times10^{-2}$, $r_2 = 23.22\times10^{-2}$, $V_0 = 46.51$, $V_1 = 6.74$, $V_2 = -117.52$, $V_E = -3.22$ and $\beta_0 = 0.5928$, the imaginary potential $W_0 = 5.293$ MeV and its energy dependence $W_E = 33.88\times10^{-2}$. The nuclear radius parameter r_0 is also fitted reasonably well to $1.378A^\gamma$ fm which means that the nuclear potential radius $R = r_0'A^{\frac{1}{3}+\gamma}$ where $\gamma = 7.93\times10^{-3}$ is a very small number (needed for fine tuning) compared to $\frac{1}{3}$.

Each calculation is performed at neutron incident energy intervals of 1 MeV and for various elements. The extracted parameters for the present analytical model provide global fits to the neutron total cross sections up to 600 MeV over quite a large number of nuclei as accurately as the phenomenological optical model potentials which are limited up to 150-200 MeV. For illustration we show only four plots here in Fig.1. The angular distribution is also well estimated at forward angles (0-30 degrees) as shown in Fig.2. We conclude that the present estimates of neutron scattering and reaction cross sections are very important for various calculations of nuclear applications[5].

References

[1] R. W. Finlay et al., Phys. Rev **C 47**, 237 (1993).
[2] W. P. Abfalterer et al., Phys. Rev **C 63**, 044608 (2001).
[3] F. S. Dietrich et al., Phys. Rev. **C 67**, 044606 (2003).
[4] Y. Wang, J. Rapaport, Nucl. Phys. **A 517**, 301 (1990).
[5] Tapan Mukhopadhyay and D. N. Basu, Phys. Rev. **C 76**, 064610 (2007); *ibid* Phys. Rev. **C 79**, 017602 (2009).