Modifications to the Mesonic Regge Trajectories

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Introduction

Understanding of hadronic spectrum has been a very important challenge in nuclear physics. It is well described by the string model of confined quarks in hadrons [1]. These strings are characterised by the Regge trajectories of hadrons (i.e., the relation between the classical mass M and spin J of hadrons).

A Regge trajectory is described by,

$$J = \alpha' M^2 + \alpha_0 \tag{1}$$

where, α_0 is a constant known as the Regge intercept and α' is the Regge slope parameter which is given by $\alpha' = 1/(2\pi K)$ here, K is the linear energy density of the string. The Eq(1) holds for the case of constant energy density K of the string [2], that is, to a linear potential of the form $V(r) = Kr$ where, r is the inter-quark distance.

The explanation of non-linearity in the Regge trajectories with physical realisation is still a challenge [3–5]. In this paper, we have analysed a more realistic string model of mesons by considering the modifications in Regge trajectories due to the finite quark mass. It is shown that the linearity of the Regge trajectories remain intact in case of mesons.

Formalism and Calculations

For hadrons with massless and spinless quarks the Regge trajectories are given by Eq(1). For a meson which is made of two different kind of massive but spinless quark-antiquark, let us assume that the quark and antiquark are sitting at the opposite ends of the string of length l. The mesonic string will rotate about its center of mass. Let the quark with mass m_1 is rotating with speed fc where $0 < f \leq 1$ and c is the speed of light in vacuum. Throughout our calculations, we have considered the natural units, i.e., $c = 1$ or, $fc = f$. Let the mass of antiquark is m_2. The modified mass of the meson (M_{mod}) is then given by

$$M_{mod} = \frac{Km_2 l}{f(m_1+m_2)} \left(\int_0^f \frac{dv}{\sqrt{1-v^2}} + \int_0^{\frac{m_1}{m_2}f} \frac{dv}{\sqrt{1-v^2}} \right) + \gamma_1 m_1 + \gamma_2 m_2 \tag{2}$$

where $\gamma_1 = \frac{1}{\sqrt{1-f^2}}$ and $\gamma_2 = \frac{1}{\sqrt{1-\frac{m_1^2 f^2}{m_2^2}}}$. After integration, Eq(2) can be rewritten as

$$M_{mod} = \frac{Km_2 l}{f(m_1+m_2)} \left(sin^{-1} f + sin^{-1} \frac{m_1 f}{m_2} \right) + \gamma_1 m_1 + \gamma_2 m_2 \tag{3}$$

The modified angular momentum of meson (J_{mod}) is given by

$$J_{mod} = \frac{Km_2^2 l^2}{f^2(m_1+m_2)^2} \left(\int_0^f \frac{v^2\, dv}{\sqrt{1-v^2}} + \int_0^{\frac{m_1}{m_2}f} \frac{dv}{\sqrt{1-v^2}} \right) + \frac{m_1 f l}{m_1+m_2} (\gamma_1 m_2 + \gamma_2 m_1) \tag{4}$$

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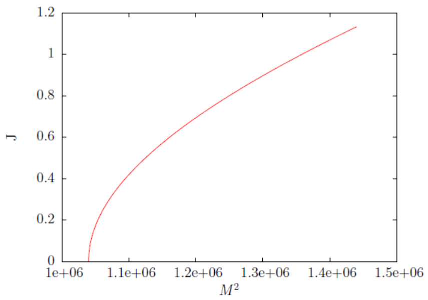

FIG. 1: The Regge trajectory for mesons when the quark '1"s speed is varying and it's mass remains same.

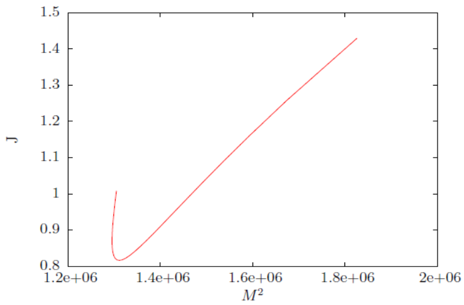

FIG. 2: The Regge trajectory for meson when the mass of quark '1' is varying and it's speed remains same.

After integration and simplification the modified mesonic Regge trajectory is obtained as

$$\begin{aligned} J_{mod} &= \frac{\pi\alpha'}{sin^{-1}f + sin^{-1}\frac{m_1 f}{m_2}} \\ &\times(M - \gamma_1 m_1 - \gamma_2 m_2)^2 \\ &\times\left(1 - \frac{sin(2sin^{-1}f) + sin\left(2sin^{-1}\frac{m_1 f}{m_2}\right)}{2\left(sin^{-1}f + sin^{-1}\frac{m_1 f}{m_2}\right)}\right) \\ &+ \frac{2\pi\left(\frac{m_1}{m_2}\right)f^2\alpha'}{sin^{-1}f + sin^{-1}\frac{m_1 f}{m_2}}(M - \gamma_1 m_1 - \gamma_2 m_2) \\ &\times(\gamma_1 m_2 + \gamma_2 m_1) \end{aligned} \quad (5)$$

The expressions of J_{mod} are functions of $sin^{-1}\frac{m_1 f}{m_2}$. Since $sin\theta \leq 1$ so $f \leq \frac{m_2}{m_1}$. Further, according to special theory of relativity $f \leq 1$. Both of these conditions should therefore satisfy simultaneously.

Discussion and Results

In computations, we have taken $m_1 = 1.5MeV(u\ quark), m_2 = 3.0MeV(d\ quark), K = 0.2GeV^2$, from Particle Data Group (PDG) [6]. For computation of mass of hadrons the string length $l = 1fm$ is taken. Fig1 shows the mesonic Regge trajectory for different quark speeds and same quark mass. In the figure in low velocity range (non-relativistic range), they show non-linear behavior. Fig2 shows that in low mass range for one value of mesonic mass two values of angular momenta are possible. In fact, these two angular momenta are for two different mesons having same mass.

Conclusions

On considering the massive quarks in the string model of hadrons, the expressions for the classical mass and classical angular momentum and hence the Regge trajectory are modified. For mesons, the form of the Regge trajectory (i.e. their linear behavior) remains same. In low mass and angular momentum region two hadrons with different quark compositions can have same mass and angular momentum.

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