Application of three-body model to study double-Λ hypernuclei using a variational approach

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Introduction

Very recently, the new experimental data have come up with the values of double-Λ binding energy $B_{\Lambda\Lambda} = 7.25 \pm 0.14$ MeV and $\Delta B_{\Lambda\Lambda} = 1.01 \pm 0.2$ MeV [1] which are substantially lower than the old data of 1966 [2]. This prompted us to study the double-Λ binding energies $B_{\Lambda\Lambda}$ anew using the earlier model [3].

The three body model

In this model, the hypernucleus is treated as a three-body system consisting of a core and the two Λ particles. The wave function for the internal motion of the three-body system is considered as

$$\Psi(r_1, r_2, r_3) = \Phi(R). \qquad (1)$$

The new space coordinate R is defined as $R = (r_1 + r_2 + \eta r_3)/2$ where r_1, r_2, r_3 are the distances between the particle pairs 1-2, 1-3 and 2-3 respectively. 1 refers to the core and 2,3 refer to the two Λ particles. Scaling parameter η controls the way the wave function depends on r_1, r_2, r_3. Defining $F(R) = R^{5/2}\Phi(R)$, the problem simplifies to an effective two-body equation in $F(R)$ along with a long range potential $V_{eff}(R)$ given by

$$\frac{d^2F}{dR^2}+\Big[\frac{4(\eta^2+5\eta+8)}{D'}E-\frac{V_{eff}(R)}{D}-\frac{15}{4R^2}\Big]F=0 \qquad (2)$$

Numerical solution of Eq.(2) gives the eigen value E and $F(R)$. The $\Lambda - \Lambda$ potential 3-G2 [4] and 3G3 [5] has been fitted with the new $B_{\Lambda\Lambda}$ value of ${}^{6}_{\Lambda\Lambda}He$ which acts as a constraint on the potential. In continuation of our recent work on double-Λ hypernuclei, we report here our study on ${}^{6}_{\Lambda\Lambda}He$, ${}^{26}_{\Lambda\Lambda}Mg$, ${}^{30}_{\Lambda\Lambda}Si$, ${}^{34}_{\Lambda\Lambda}S$, ${}^{40}_{\Lambda\Lambda}Ar$. The core is even-even for all the double-Λ hypernuclei listed here. The Λ-separation energy B_Λ, has been calculated using the empirical formula

$$B_\Lambda(A) = 27.0 - 81.9A^{-2/3} \pm 1.5. \qquad (3)$$

where A is mass number of the core. The core-Λ potential is considered to be Woods-Saxon

$$V_{core-\Lambda} = -\frac{V_0}{1+\exp(\frac{r-c}{a})} \qquad (4)$$

where c=$r_0A^{1/3}$, r_0=1.128+0.439$A^{-2/3}$ fm.

Calculations and Results

Using this potential in the Schrodinger's equation for the core and the Λ particle, the differential equation is numerically solved to determine the Λ-separation energy. V_0 and a are adjusted till the Λ-separation energy so obtained tallies with the value determined from the empirical relation of Eq.(3). The values of V_0 and a thus obtained lie within the limits of the best fit values predicted by the RMF calculation [6]. Core mass is calculated using the relation

$$\text{Mass excess} = (\text{Mass} - A)\chi_0 \qquad (5)$$

where χ_0= (93148±5) keV and mass excess is tabulated in mass tables. With the core-Λ and the Λ-Λ potentials η is varied to get E_{min}. Here $B_{\Lambda\Lambda} = -E_{min}$. We have achieved convergence in the two-Λ separation energy $B_{\Lambda\Lambda}$ for each hypernucleus. From the function $F(R)$, the r.m.s values of core-Λ separation $< r_{core-\Lambda} >$ and the Λ-Λ separation $< r_{\Lambda\Lambda} >$ are calculated. The results are given in the Table-1.

Summary and Conclusion

In summary, with regard to finding double-Λ binding energies $B_{\Lambda\Lambda}$, the present approach

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TABLE I: Results of the present calculations for double-Λ hypernuclei.

double-Λ hypernuclei	B_Λ (MeV)	Λ-Λ (Potn.)	$B_{\Lambda\Lambda}$ (MeV)	$< r_{core-\Lambda}$ (fm)	$< r_{\Lambda\Lambda} >$ (fm)	H-H method [7] B_Λ	H-H method [7] $B_{\Lambda\Lambda}$
${}^{6}_{\Lambda\Lambda}He$	2.94	3G2	7.476	2.815	3.353	3.12	10.80
		3G3	7.425	2.787	3.204		
${}^{26}_{\Lambda\Lambda}Mg$	17.403	3G2	36.635	2.312	2.894	17.423	40.59
		3G3	36.739	2.281	2.814		
${}^{30}_{\Lambda\Lambda}Si$	18.325	3G2	38.274	2.358	2.923	18.33	42.19
		3G3	38.394	2.331	2.825		
${}^{34}_{\Lambda\Lambda}S$	19.041	3G2	39.567	2.387	2.89	19.04	43.43
		3G3	39.674	2.376	2.88		
${}^{40}_{\Lambda\Lambda}Ar$	19.875	3G2	41.130	2.581	2.98	- - - -	- - - -
		3G3	41.243	2.441	2.959		

appears to work well over a wide mass range of double-Λ hypernuclei with even-even cores.

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