Charmonium spectra and decay widths using Hulthen potential

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Introduction

The study of heavy quarkonium systems has played an important role in the development of quantum chromodynamics (QCD).The investigation of the properties of these mesons gives important information about heavy quark antiquark interactions and heavy quark dynamics. The prediction of the properties of strongly interacting particles using the mesonic and baryonic sectors is of great importance since these particles can be easily produced in experiments and hence the theoretical predictions can be verified. Since the exact form of the QCD potential is not known, one has to use phenomenological models to predict the hadronic properties. The phenemenological models are either nonrelativistic [1–3] or relativistic [4, 5]. The Hamiltonian of these quark models usually contain three main ingredients: the kinetic energy, the confinement potential and a hyperfine interaction term. Using the quark model the hadron spectra have been predicted successfully. The prediction of mass spectrum alone doesn't guarantee the validity of a model for describing hadronic interactions. This is because different potentials have been proposed which reproduce the same spectra. Therefore using the model, one must be able to calculate other observables like the radiative decay widths, the leptonic decay widths, the two-photon decay widths, etc. In this article, we calculate the charmonium spectra in the general framework of a nonrelativistic potential model. The full $Q\bar{Q}$ potential used in our model consists of a Hulthen potential and a confining linear potential. The model parameters and the wavefunction that reproduce the mass spectra are used to study their decay properties.

Theoretical Model

In the present work for the study of the heavy quark bound systems, we have considered the following nonrelativistic Hamiltonian,

$$H = M + \frac{p^2}{2\mu} + V(r). \tag{1}$$

We have considered a potential of the form [6],

$$V(r) = V_H(r) + V_C(r) + V_0, \tag{2}$$

where V_H is the Hulthen potential [7],

$$V_H(r) = -\mu_0 \frac{\exp(-r/\mu)}{1 - \exp(-r/\mu)}, \tag{3}$$

The confining part of the potential V_C is given by,

$$V_C(r) = cr, \tag{4}$$

In (2), V_0 is a constant. To the Hamiltonian (1), we add separately the spin-spin, the spin-orbit and the tensor interaction terms.

In our work, we have used the three dimensional harmonic oscillator wavefunction as the trial wavefunction for obtaining the $Q\bar{Q}$ mass spectrum. In order to obtain the $Q\bar{Q}$ spectrum, we have solved the Schrodinger equation with the Hamiltonian Eq. (1) using the variational method. The model parameters and the wavefunction that reproduce the mass spectra are used to investigate the decay constants, leptonic decay widths, two-photon and two-gluon decay widths, and radiative decay widths of the $c\bar{c}$ mesons.

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The leptonic decay width for a ${}^3S_1$ state to decay into two leptons is given by the Van Royen and Weisskopf relation [8],

$$\bar{\Gamma}_{ee_-} = \frac{16\pi\alpha^2 e_Q^2 |\Psi(0)|^2}{m_n^2} \times (1 - 16\alpha_s/3\pi),$$

where $\alpha(= 1/137)$ is the electromagnetic fine structure constant, e_Q is the quark charge, m_n is the mass of the $n\ {}^3S_1$ state, and $|\Psi(0)|$ is the value of the relative $n\ {}^3S_1$ wavefunction at the origin. The term in the parenthesis is the first order QCD correction factor. The partial widths for electric dipole (E1) transition ${}^{2S+1}L_{i\,J_i} \rightarrow {}^{2S+1}L_{f\,J_f} + \gamma$ is given by [8]:

$$\Gamma = \frac{4\alpha e_Q^2 k^3}{3}(2J_f + 1)S_{if}|\langle f|r|i\rangle|^2,$$

where the statistical factor,

$$S_{if} = S_{fi} = \max(L_i, L_f)\left\{\begin{array}{ccc} J_i & 1 & J_f \\ L_f & S & L_i \end{array}\right\}^2$$

The photon energy k is given by,

$$k = \frac{m_f^2 - m_i^2}{2m_f}. \qquad (5)$$

Here, m_i and m_f are the initial and final state masses of the quarkonia respectively.

Results and discussions

TABLE I: Mass Spectrum (in MeV)

Meson	Present	Exp.[9]	[10]	[5]	[11]
J/ψ	3097	3097	3096	3096	3139
$\psi(2S)$	3689	3686	3690	3686	3694
$\psi(3S)$	4140	4039	4082	4088	4085
$\psi(4S)$	4532	4421	4408		4412
$\eta_c(1S)$	2980	2980	2985	2979	3052
$\eta_c(2S)$	3600	3637	3626	3588	3655
$h_c(1P)$	3506	3525	3497	3526	3529
$\chi_{c0}(1P)$	3413	3414	3431	3424	3435
$\chi_{c1}(1P)$	3496	3510	3464	3510	3511
$\chi_{c2}(1P)$	3576	3556	3530	3556	3588

In the present work, using a $Q\bar{Q}$ interaction, which consists of a Hulthen potential and a linear potential, the spectra and decay widths of $c\bar{c}$ mesons were computed. The results obtained agree with the experimental results and with the predictions from other theoretical models. The results are tabulated in Table I and Table II.

TABLE II: E1 radiative decay widths (in MeV)

Decay	Present	Exp.	[5]
$\psi(2S) \rightarrow \chi_{c0}(1P) + \gamma$	27	25.8	26
$\psi(2S) \rightarrow \chi_{c1}(1P) + \gamma$	29	24.1	23
$\psi(2S) \rightarrow \chi_{c2}(1P) + \gamma$	10	21.6	18
$\eta_c(2S) \rightarrow h_c(1P) + \gamma$	10		6
$\chi_{c0}(1P) \rightarrow J/\psi(1S) + \gamma$	182	121.7	121
$\chi_{c1}(1P) \rightarrow J/\psi(1S) + \gamma$	354	295.8	265
$\chi_{c2}(1P) \rightarrow J/\psi(1S) + \gamma$	590	384.1	327

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