Spectrum and decays of $b\bar{b}$ mesons in a nonrelativistic model

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Introduction

The recent discoveries of conventional states $h_c(1P)$, $\chi_{c2}(2P)$, $\eta_b(1S)$, $\eta_c(2S)$ and the observation of the unanticipated states like $X(3872)$, $X(3915)$, $Y(4260)$, $Z(3930)$, etc [1] have led to a renewed interest in quarkonium systems. Quarkonium consists of a heavy quark and its anti-quark, ($c\bar{c}$ or $b\bar{b}$). Since the exact form of the QCD potential is not available, one has to use phenomenological models to predict the hadronic properties. These models are either nonrelativistic [2–4] or relativistic [5, 6]. The Hamiltonian of these quark models usually contain three main ingredients: the kinetic energy, the confinement potential and a hyperfine interaction term. The nonrelativistic quark model (NRQM) have been proven to be very successful in describing hadronic properties. In this present work, we calculate the S, P and D wave bottomonium ($b\bar{b}$) spectra using a nonrelativistic potential model. The full $Q\bar{Q}$ potential used in our model consists of a Hulthen potential and a confining linear potential. The model parameters and the wavefunction that reproduce the mass spectra are used to study their decay properties.

Theoretical Model

In the present work for the study of the heavy quark bound systems, we have considered the following nonrelativistic Hamiltonian,

$$H = M + \frac{p^2}{2\mu} + V(r). \tag{1}$$

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We have considered a potential of the form [7],

$$V(r) = V_H(r) + V_C(r) + V_0, \tag{2}$$

where V_H is the Hulthen potential [8],

$$V_H(r) = -\mu_0 \frac{\exp(-r/\mu)}{1-\exp(-r/\mu)}, \tag{3}$$

The confining part of the potential V_C is given by,

$$V_C(r) = cr, \tag{4}$$

In (2), V_0 is a constant. To the Hamiltonian (1), we add separately the spin-spin, the spin-orbit and the tensor interaction terms [9, 10]:

$$\begin{aligned} H_{SS} &= \frac{2}{3m^2}\vec{S}_Q\cdot\vec{S}_{\bar{Q}}\ \Delta V_v(r), \\ H_{LS} &= \frac{1}{2m^2r}\vec{L}\cdot\vec{S}\ \left[3\frac{d}{dr}V_v(r) - \frac{d}{dr}V_s(r)\right], \\ H_T &= \frac{1}{12m^2}S_{12}\ \left[\frac{1}{r}\frac{d}{dr}V_v(r) - \frac{d^2}{dr^2}V_v(r)\right], \end{aligned}$$

where the spin dependent factor,

$$S_{12} = 2\left[3\frac{(\vec{S}\cdot\hat{r})^2}{r^2} - \vec{S}^2\right].$$

In our work, we have used the three dimensional harmonic oscillator wavefunction as the trial wavefunction for obtaining the $Q\bar{Q}$ mass spectrum. In order to obtain the $Q\bar{Q}$ spectrum, we have solved the Schrodinger equation with the Hamiltonian Eq. (1) using the variational method. The model parameters and the wavefunction that reproduce the mass spectra are used to investigate the decay constants, leptonic decay widths, two-photon and two-gluon decay widths, and radiative decay widths of the $b\bar{b}$ mesons.

TABLE I: Mass Spectrum (in MeV)

Meson	Present	Exp.[11]	[12]	[6]	[13]
$\Upsilon(1S)$	9460	9460	9458	9460	9460
$\Upsilon(2S)$	9981	10023	9950	10023	10020
$\Upsilon(3S)$	10319	10355		10355	10390
$\Upsilon(4S)$	10599	10579			10680
$\Upsilon(5S)$	10846	10865			10930
$\eta_b(1S)$	9390	9391	9395	9400	9410
$\eta_b(2S)$	9947		9773	9993	10000
$h_b(1P)$	9891	9898	9932	9901	9886
$\chi_{b0}(1P)$	9863	9859	9909	9863	9850
$\chi_{b1}(1P)$	9886	9892	9929	9892	9870
$\chi_{b2}(1P)$	9907	9912	9938	9913	9890
$h_b(2P)$	10246	10259	10210	10261	10270
$\chi_{b0}(2P)$	10222	10232	10130	10234	10240
$\chi_{b1}(2P)$	10242	10255	10208	10255	10260
$\chi_{b2}(2P)$	10259	10269	10214	10268	10280
$1\ ^1D_2$	10137		10252	10158	10150
$1\ ^3D_1$	10122		10253	10153	10140
$1\ ^3D_2$	10136	10162	10254	10158	10150
$1\ ^3D_3$	10149		10250	10162	10150

TABLE II: Decay Widths

Decay	Present	[14]
$\Upsilon(1S) \to \gamma\gamma$	0.42 keV	0.55 keV
$\Upsilon(2S) \to \gamma\gamma$	0.20 keV	0.20 keV
$\Upsilon(3S) \to \gamma\gamma$	0.17 keV	0.22 keV
$\Upsilon(1S) \to gg$	18.80 MeV	11.49 MeV
$\Upsilon(2S) \to gg$	9.22 MeV	5.16 MeV
$\Upsilon(3S) \to gg$	7.64 MeV	3.80 MeV

Results and discussions

In this article we have done a comprehensive study of the S, P and D wave $b\bar{b}$ spectrum and its decays. The $Q\bar{Q}$ interaction used in our analysis consists of a Hulthen potential and a linear potential. The spectra and decay widths of $b\bar{b}$ mesons thus obtained agree with the experimental results and with the predictions from other theoretical models. The results are tabulated in Table I and Table II.

References

[1] Brambilla N *et al.* 2011 arXiv: hep-ph/1010.5827

[2] D. Gromes Nucl. Phys. B **130** (1977) 18.

[3] O. W. Greenberg Ann. Rev. Nucl. Part. Sci. **28** (1978) 327.

[4] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane and T. M. Yan, Phys. Rev. D **17** (1978) 3090.

[5] K. B. Vijayakumar, B. Hanumaiah and S. Pepin Eur. Phys. J. A **19** (2004) 247.

[6] D. Ebert, R. N. Faustov and V. O. Galkin Phys. Rev. D **67** (2003) 014027.

[7] Bhaghyesh, K. B. Vijaya Kumar, Antony Prakash Monteiro, J. Phys. G: Nucl. Part. Phys. 38 (2011) 085001.

[8] L. Hulthen, Ark. Mat. Astron. Fys. A **28** (1942) 5.

[9] Wolfgang Lucha, Franz F. Schöberl, (1996) arXiv:9601263v1 [hep-ph].

[10] M.B. Voloshin, Prog. Part. Nucl. Phys. 61 (2008) 455, arXiv:0711.4556v3 [hep-ph].

[11] K. Nakamura, et al., Particle Data Group, J. Phys. G: Nucl. Part. Phys. **37** (2010) 075021.

[12] Arpit Parmar, Bhavin Patel, P C Vinodkumar, Nucl. Phys. A 848 (2010) 299, arXiv:1001.0848v2 [hep-ph].

[13] J. Zeng, J. W. Van Orden, W. Roberts, Phys. Rev. D 52 (1995) 5229.

[14] Laverty J T, Radford S F and Repko W W 2009 arXiv: hep-ph/0901.3917v3