Δ contributions to $pp \to pp\pi^0$

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Introduction

Ever since the total crosssection measurements [1] for $pp \to pp\pi^0$ were found to be more than factor of 5 larger than the then available theoretical prediction [2], experimental and theoretical study of the reaction excited considerable interest. Advances in storage ring technology led to detailed experimental studies including measurements of spin observables [3] when both the colliding protons are polarised. The Julich meson exchange model [4] was thoroughly confronted with this data. It was found that the model was comparitively more successful with the less complete data [5] on $\vec{p}\vec{p} \to d\pi^+$ and $\vec{p}\vec{p} \to pn\pi^+$ but fail to provide an overall satisfactory reproduction of the data on $pp \to pp\pi^0$. In this context, a model independent approach to the reaction was developed [6] using irreducible tensor techniques [7]. This appproach was employed [8] to incisively analyse the findings of the Julich model, which revealed that (i)the Δ contributions are important (ii) the model deviated very strongly in the case of ${}^3P_1 \to {}^3P_0 p$ and to a lesser extent in ${}^3F_3 \to {}^3P_2 p$. Because the final spin singlet and spin triplet states do not mix in any of the spin observables measured in [3], both these amplitudes were assumed to be real in [8]. It was shown more recently [9] how this drawback can be removed.

The purpose of the present paper is to discuss the Δ contributions to $pp \to pp\pi^0$ employing the model independent approach.

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Theoretical Formalism

If $\mathbf{p_1}, \mathbf{p_2}, \mathbf{q}$ denote respectively the *c.m.* momenta of the 2 nucleons and the pion in the final state and $\mathbf{p_i}$ the *c.m.* momentum in the initial state, we may define invariant masses W_1 and W_2 for the two $\pi^0 - p$ systems in the final state and choose events corresponding to $W_1, W_2 = m_\Delta$ for the analysis, where m_Δ denotes the mass of Δ resonance. The matrix M for $pp \to p\Delta$ is given by

$$M = \sum_{s_i=0}^{1} \sum_{s_f=1}^{2} \sum_{\lambda=|s_f-s_i|}^{|s_f+s_i|} \left(S^\lambda(s_f, s_i).M^\lambda(s_f, s_i)\right) , \quad (1)$$

where the irreducible tensor amplitudes are

$$M^\lambda_\mu(s_f, s_i) = \sum_\alpha G_\alpha f_\alpha (Y_{l_f}(\hat{\mathbf{p_f}}) \otimes Y_{l_i}(\hat{\mathbf{p_i}}))^\lambda_\mu, \quad (2)$$

in terms of the partial wave amplitudes $f_\alpha = M^j_{l_f s_f l_i s_i}(E)$ at any given *c.m.* energy E and the geometrical factors $G_\alpha = (-1)^{l_i+s_i+l_f-j}[j]^2[s_f]^{-1}W(s_i l_i s_f l_f; j\lambda)$. The final *c.m.* momentum $\mathbf{p_f} = \mathbf{p_1} + \mathbf{q} - \mathbf{p_2}$, if $W_1 = m_\Delta$ and $\mathbf{p_f} = \mathbf{p_2} + \mathbf{q} - \mathbf{p_1}$, if $W_2 = m_\Delta$. Conservation of iso-spin I implies that I can take only one value $I = 1$ and $l_i + s_i$ must be even because of Pauli exclusion principle. Parity conservation implies that $(-1)^{l_i} = (-1)^{l_f}$. Limiting ourselves to $l_f = 0, 1$ at threshold energies, we have a set of 9 partial

TABLE I: Threshold Partial Wave Amplitudes

	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9
l_f	0	1	1	1	1	1	1	1	1
s_f	2	1	1	1	1	2	2	2	2
j	2	0	1	2	2	1	2	2	3
l_i	2	1	1	1	3	1	1	3	3
s_i	0	1	1	1	1	1	1	1	1

wave amplitudes f_α as shown in table 1, in terms of which the non-zero irreducible tensor amplitudes are given by

$$\begin{aligned}
M^0_0(1,1) &= g_2\cos\theta_f\,,\\
M^2_0(1,1) &= \frac{-4\sqrt{2}}{\sqrt{3}}(g_4-3g_5)\cos\theta_f\,,\\
M^1_{\pm1}(1,1) &= g_3\sin\theta_f\,,\\
M^2_{\pm1}(1,1) &= \pm(g_4+2g_5)\sin\theta_f\,,\\
M^2_0(2,0) &= g_1\,,\\
M^1_{\pm1}(2,1) &= g_6\sin\theta_f\,,\\
M^2_0(2,1) &= (g_8+g_9)\cos\theta_f\,,\\
M^2_{\pm1}(2,1) &= \mp\frac{1}{2\sqrt{6}}[3g_8+2g_9]\sin\theta_f\,,\\
M^3_{\pm1}(2,1) &= g_7\sin\theta_f\,, \qquad (3)
\end{aligned}$$

where $g_\beta, \beta = 1,2,....,9$ are precisely determined and invertible linear combinations of the $f_\alpha, \alpha = 1,2,....,9$ in our model independent theoretical approach. Apart from the differential cross section $\frac{d\sigma_0}{d\Omega}$ with initially unpolarized protons one may measure the state of polarization of the Δ resonance which is specified by Fano-statistical tensors

$$\begin{aligned}
t^k_q &= \sum_{\lambda,\lambda',s_i,s_f,s'_f}(-1)^{s'_f-s_i}[\lambda][\lambda']^{-1}B^k_q\\
&\quad\times W(s'_f\lambda' s_f\lambda; s_i k)\,, \qquad (4)
\end{aligned}$$

where $t^0_0 = \frac{d\sigma_0}{d\Omega}$ and

$$B^k_q = (M^\lambda(s_f,s_i)\otimes M^{\dagger\lambda'}(s'_f,s'_i))^k_q\,, \quad (5)$$

with k taking values $0,1,2,3$ and $q = -k\cdots k$. The t^k_q by themselves constitute a set of 16 observables, bilinear in terms of the irreducible tensor amplitudes. In an experiment like [3] with initially polarized protons the differential cross-section is given by

$$\begin{aligned}
\frac{d\sigma}{d\Omega} &= \sum_{\lambda,\lambda',s_i,s'_i,s_f,s'_f,k}(-1)^\lambda\frac{[\lambda][s_i]}{[\lambda']}\\
&\quad\times W(s'_i k s_f\lambda; s_i\lambda')(I^k.B^k)\,, \qquad (6)
\end{aligned}$$

which may be expressed in the standard form

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_0}{d\Omega}\left[1+\sum_{k_i}\left(I^k.A^k\right)\right]\,, \qquad (7)$$

where the Fano-statistical tensors $I^k_q(s_i,s'_i)$ characterize the initial $\vec{p}-\vec{p}$ system and A^k_q are the analysing powers.
It should therefore be feasible to measure experimentally the Fano-statistical tensors t^k_q and the analysing powers A^k_q to determine empirically the threshold partial wave amplitudes $f_1, f_2\cdots f_9$ and hence assess the importance of the Δ contributions.

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