S-wave Properties of Bottonium using Martin like Potential

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Introduction

The recent discovery of the η_b $(1S)$ states [1] and many high precision experimental observations of various hadronic states [2] have necessitated reconsideration of the parameters involved in the previously employed phenomenological models to study the hadrons and to improve their predictability. In this context, we consider the bottonium states and study their mass spectra and compute the decay properties such as the leptonic and di-gamma decay widths of few lowlying s-wave $b\bar{b}$ states with no additional parameters.

Methodology

It has been shown that a purely phenological approach to the nonrelativistic potential-model study of Υ spectra can lead to a static non-Coulombic Power-law potential of the form [3, 4]

$$V(r) = \lambda r^{\nu} + V_0 \quad (1)$$

where ν is close to 0.1 and $A > 0$.

Following general quantum mechanical rules as discussed in [5], the binding energy of a system with reduced mass μ in a power law potential, λr^{ν} is given by

$$E_{nl} = \lambda^{2/(2+\nu)} \quad (2\mu)^{-\nu/(2+\nu)} \left[A(\nu)\left(n+\frac{l}{2}-\frac{1}{4}\right)\right]^{2\nu/(2+\nu)} \quad (2)$$

and the corresponding square of the probability amplitude of the s-waves at the origin is given by

$$|\psi_n(0)|^2 = \frac{1}{2\pi^2}\left(\frac{2\mu\lambda}{\hbar^2}\right)^{3/(2+\nu)} \frac{\nu}{(2+\nu)} [A(\nu)]^{3\nu/(2+\nu)}\left(n-\frac{1}{4}\right)^{2(\nu-1)/(2+\nu)} \quad (3)$$

where

$$A(\nu) = \left[2\nu\sqrt{\pi}\,\Gamma\left(\frac{3}{2}+\frac{1}{\nu}\right)\right]/\Gamma(1/\nu), \quad \nu > 0. \quad (4)$$

The nonrelativistic Schrodinger bound-state mass (spin average mass) of the $Q\bar{Q}$ system follows as

$$M_{SA} = 2m_b + V_0 + E_{nl} \quad (5)$$

For the hyperfine split we have considered the standard one gluon exchange interaction [6]. Accordingly,the hyperfine mass split for the s-wave is given by

$$\Delta M = A_{hyp}|\psi_n(0)|^2/m_b^2. \quad (6)$$

The b-quark mass, m_b is taken as 4.67 GeV as given in PDG [2]. The vector $(\Upsilon(nS))$ and the pseudoscalar $(\eta_b(nS))$ masses are obtained by adding $\Delta M/4$ and $-3\Delta M/4$ respectively to the corresponding spin average mass of the nS state given by eq.(5). A fit to this mass formula using the experimental masses of $\Upsilon(1S, 2S)$ and the newly discovered $\eta_b(1S)$ states provides us the potential parameters λ, V_0 and the hyperfine parameter (A_{hyp}). The predicted $\Upsilon(nS)$ and $\eta_b(nS)$ states are presented in Table I.

TABLE I: Results for $b\bar{b}$ spectrum using martin like potentials $V(r) = V_0 + \lambda r^\nu$

nS	M_V (GeV)	M_V (GeV) [7]	M_V (GeV) Exp.[2]	M_P (GeV)	M_P (GeV) [7]	M_P (GeV) Exp.[2]
$1S$	**9.460**	9.460	9.460	**9.391**	9.392	9.391
$2S$	**10.023**	10.023	10.023	9.989	9.987	-
$3S$	10.350	10.364	10.355	10.327	10.333	-
$4S$	10.581	10.636	10.579	10.564	10.609	-

TABLE II: The leptonic widths of the $\Upsilon(nS)$ and the di-gamma widths of $\eta_b(nS)$ states

nS	$\Gamma^{l^+l^-}$ (keV)	$\Gamma^{l^+l^-}$ (keV) [7]	$\Gamma^{l^+l^-}$ (keV) Exp.[2]	$\Gamma^{\gamma\gamma}$ (keV)
$1S$	1.353	1.33	1.34±0.018	0.1774
$2S$	0.578	0.62	0.612±0.011	0.0750
$3S$	0.373	0.48	0.443±0.008	0.0482
$4S$	0.274	0.40	0.272±0.029	0.0353

Leptonic and di-gamma decay widths of Bottonium states

The leptonic decay width with the radiative correction of $\Upsilon(nS) \to l^+l^-$ is computed as [8–10]

$$\Gamma^{l^+l^-} = \frac{4\alpha_e^2 e_Q^2}{M_V^2}|\psi_{nl}^2(0)|\left[1 - \frac{16}{3\pi}\alpha_s\right] \qquad (7)$$

And the di-gamma (two photon) decay width of $\eta_b(nS) \to \gamma\gamma$ states with radiative correction is obtained as

$$\Gamma^{\gamma\gamma} = \frac{12\alpha_e^2 e_Q^4}{M_V^2}\ |\psi_n^2(0)|\left[1 - \frac{\alpha_s}{\pi}\left(\frac{20-\pi^2}{3}\right)\right] \qquad (8)$$

The predicted results are tabulated in Table II.

Results and discussion

We have been able to predict the bottonium s-wave masses of $(\Upsilon(3S-4S))$ states which are in good agreement with the reported PDG values as compared to the predicted values of [7]. we have also predicted the $(\eta_b(2S-4S))$ states within the mass range 9.989 GeV to 10.564 GeV. We hope to find future experimental support in favor of our predictions. With no additional parameters we have been able to predict the leptonic decay widths of $(\Upsilon(1S-4S))$ states which are in good agreement with the experimental values [2] and with other model predictions [7]. The predicted di-gamma decay widths of $(\eta_b(1S-4S))$ states would be helpful to identify the $(\eta_b(nS))$ resonances in future experiments.

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