P-wave masses of the D_s Meson

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Introduction

For the charm-strange meson the P-wave states with quantum numbers $0^+(D^\star_{s0}(2317))$, $1^+(D_{s1}(2460)$ and $D'_{s1}(2536))$ and $2^+(D^\star_{s2}(2573))$ are very well established experimentally[1].

Masses of these states along with other properties have been studied in many theoretical schemes[2–4]. For the P-wave masses one finds that there is considerable spread from the experimental values. In particular the $D^\star_{s0}(2317)$ and $D_{s1}(2460)$ states are predicted to be heavier than experimental observations.

Since this is a heavy-light bound system it will be interesting to see how far one can extend the usual non-relativistic potential model formalism. With this in perspective in this paper we study the D_s meson in a variational scheme employing the Hydrogenic wave function as a trial wave function. We include the kinematic relativistic corrections to the Hamiltonian and also the spin-spin, spin-orbit and tensor interactions to obtain the P-wave masses.

Theoretical framework

The Hamiltonian for D_s meson could be written as[3, 5]

$$H = \sqrt{\mathbf{p}^2 + m_Q^2} + \sqrt{\mathbf{p}^2 + m_{\bar{q}}^2} + V(\mathbf{r}) \quad (1)$$

where $\mathbf{p}$ is the relative momentum of the quark-antiquark and m_Q is the heavy quark mass and $m_{\bar{q}}$ is the light quark mass. The Hamiltonian in Eq(1) represents the energy of the meson in the meson rest frame. We expand the kinetic energy(K.E.) part of the Hamiltonian up to $\mathcal{O}(p^6)$, and $V(\mathbf{r})$ is the quark-antiquark potential [6–8],

$$V(r) = -\frac{\alpha_c}{r} + Ar^\nu + V_0 \quad (2)$$

where A is the potential parameter and ν is a general power index with a constant term V_0.

We have used the hydrogenic radial wave function in the present study. The hydrogenic wave function has the form

$$R_{nl}(r) = \left(\frac{\mu^3(n-l-1)!}{2n(n+l)!}\right)^{1/2} (\mu\, r)^l \times e^{-\mu r/2}\, L^{2l+1}_{n-l-1}(\mu r) \quad (3)$$

Here, μ is the variational parameter and L is Laguerre polynomial.

We obtain the expectation values of the Hamiltonian as

$$H\psi = E\psi \quad (4)$$

For a chosen value of ν, the variational parameter, μ is determined for each state using the Virial theorem

$$\langle K.E.\rangle = \frac{1}{2}\left\langle \frac{rdV}{dr}\right\rangle \quad (5)$$

As the interaction potential assumed here does not contain the spin dependent part, Eq(4) gives the spin averaged masses of the system in terms of the power index ν. The spin averaged mass for the ground state is computed for the values of ν from 0.5 to 2.0. The spin-averaged mass is matched with the experimental value for the ground state using the equation

$$M_{SA} = M_P + \frac{3}{4}(M_V - M_P) \quad (6)$$

where M_V and M_P are the vector and pseudoscalar meson masses in the ground state. This allows us to fix the value of A for a

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TABLE I: P-wave masses of D_s meson(in GeV)

ν	State			
	$D_{s0}(^3P_0)$	D'_{s1}	D_{s1}	$D^{\star}_{s2}$
0.5	2.268	2.301	2.300	2.312
1.0	2.355	2.442	2.426	2.458
1.5	2.383	2.534	2.510	2.566
2.0	2.374	2.587	2.572	2.651
Exp.[1]	2.318	2.535	2.460	2.573
GJ[3]	2.387	2.536	2.521	2.573
EFG[2]	2.509	2.574	2.536	2.571
De-min[4]	2.344	2.510	2.488	2.559

chosen ν. We add separately (in Eq.(4)) the spin-dependent part of the usual one gluon exchange potential (OGEP) between the quark anti quark for computing the hyperfine and spin-orbit shifting of the low-lying S, P and D-states. Thus to take into account the spin dependent and spin-orbit interaction, causing the splitting of the nL levels one introduces additional term in the Hamiltonian[9]

$$\begin{aligned} V_{SD}(\mathbf{r}) &= \left(\frac{\mathbf{L}\cdot\mathbf{S_Q}}{2m_Q^2}+\frac{\mathbf{L}\cdot\mathbf{s_{\bar{q}}}}{2m_{\bar{q}}^2}\right)\left(-\frac{dV(r)}{dr}+\right. \\ &\left.\frac{8}{3}\alpha_S\frac{1}{r^3}\right)+\frac{4}{3}\alpha_S\frac{1}{m_Q m_{\bar{q}}}\frac{\mathbf{L}\cdot\mathbf{S}}{r^3} \\ &+\frac{4}{3}\alpha_S\frac{2}{3m_Q m_{\bar{q}}}\mathbf{S_Q}\cdot\mathbf{s_{\bar{q}}}4\pi\delta(\mathbf{r})+ \\ &\frac{4}{3}\alpha_S\frac{1}{m_Q m_{\bar{q}}}(3(\mathbf{S_Q}\cdot\mathbf{n})-(\mathbf{s_{\bar{q}}}\cdot\mathbf{n}) \\ &-\mathbf{S_Q}\cdot\mathbf{s_{\bar{q}}})\frac{1}{r^3},\quad \mathbf{n}=\frac{\mathbf{r}}{r} \end{aligned} \tag{7}$$

where $V(r)$ is the phenomenological potential, the first terms takes into account the relativistic corrections to the potential $V(r)$, the second term accounts spin orbital interaction, third term is usual spin-spin interaction part which is responsible for pseudoscalar and vector meson splitting and fourth term stands for tensor interaction.

The input parameters used for various calculations are $m_c = 1.31\ GeV$, $m_s = 0.42\ GeV$, $V_0 = -0.075\ GeV$, $\alpha_s = 0.391$ and the fitted value of A were found to be 0.204, 0.100, 0.049, 0.023 for ν=0.5, 1.0, 1.5, and 2.0 respectively.

Conclusion

Looking to Table - I, various theoretical predictions are listed but none of these models are able to explain the 1P states satisfactorily. Results of ref[3] for the said states are close to experimental values. Masses of these states in our calculation are close to the experimental values and other theoretical predictions.

It can be seen from the table that our model successfully predicts the P-wave masses at $\nu = 1.5$. This value of index ν significantly higher than that for a linear potential which is strongly favored by lattice simulations. In conclusion we find that the Hydrogenic wave function can be used successfully provided one uses a non-linear potential with $\nu = 1.5$.

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