Spin Dependent Spectroscopy of Heavy Quarkonium

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In the present work mass spectroscopy of charmonium and bottonium systems has been studied using energy dependent quark interquark potential in the framework of non-relativistic Schrödinger wave equation. Energy dependence gives rise to nonlocality in the potential. Such potential were first of all used by Lombard et al.[1] to study mass spectroscopy of heavy quarkonium system These authors have used the interqaurk potential to be of the form of harmonic oscillator with a small linear energy dependent perturbation. Their main conclusion is that energy dependence can account for saturation of the energy levels at higher excitation energies, a feature that is observed experimentally. Main drawback of the Lombard potential is that it does not have any asymptotic form. It is well known that any general form of quark interquark potential should have an asymptotic term and a confinement term. An asymptotic term is important to account for the low energy spectrum and the short range behavior of the wave function. In view of this in the present work we have used more general energy dependent potential of the following form

$$V(r,E_{n,l}) = \frac{1}{2}m\omega^2 r^2(1+\gamma E_{n,l}) + \frac{g}{r^2} \qquad (1)$$

The particular form of the potential has special advantage that it admits exact solution of radial Schrödinger equation. The equation of eigen value equation is

$$E_{n,l} = -\frac{1}{8}a^2\omega^2\gamma + \frac{a\omega}{8}\sqrt{a^2\omega^2\gamma^2+16} \qquad (2)$$

where

$a = 4n - 2 + \sqrt{(2l+1)^2 + 8\mu g}$ and μ is the reduced mass

It has three parameters. These parameters ω, γ and g are obtained by fitting the calculated values of E_{2S}, E_{3S} and E_{1P} with respect to E_{1S}. The energy dependence leads to different potential for each state as shown in **fig1** for charmonium.

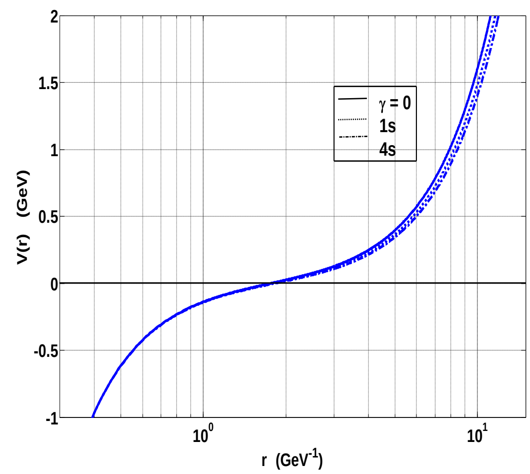

Fig 1. Quark interquark potential curves as a function of r for energy dependent (1S and 4S states) and energy independent case (γ =0) for $c\bar{c}$.

In order to obtain full spin dependent spectra Briet -Fermi correction term is added to the potential where

$$V_{BF}(r) = V_{LS}(r)(\vec{L}.\vec{S}) + V_T(r)\left[S(S+1) - \frac{3(\vec{S}.\vec{r})(\vec{S}.\vec{r})}{r^2}\right] + V_{SS}(r)\left[S(S+1) - \frac{3}{2}\right]$$

Here $V_{LS}(r)$ is the spin orbit term, $V_T(r)$, the tensor term and $V_{SS}(r)$ represents spin spin term derived by vector and scalar parts of the potential.

Using this complete spin dependent potential reduced radial wave equation has become solvable in Mathematica 8.0 by software program obtained by Lucha et al [2] for each quantum state separately. Masses of charm and bottom quark are taken as 1.5 GeV and 5.0 GeV respectively. The results of our calculation are shown in **fig 1 and 2** and compared with experimental values given in PDG 2010 [3].

The parameter χ which characterize ratio of P wave splitting is given by [4]

$$\chi = \frac{M(^3P_2) - M(^3P_1)}{M(^3P_1) - M(^3P_0)} \qquad (3)$$

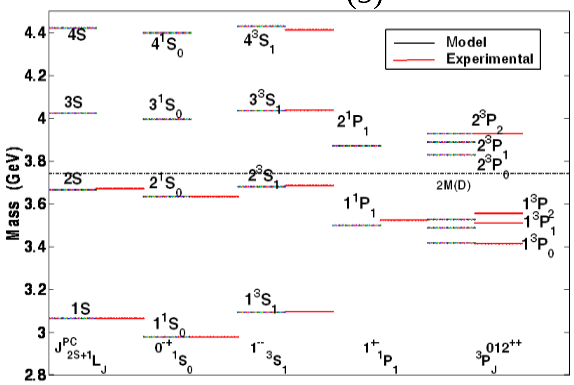

FIG1: Spin dependent mass spectrum of $c\bar{c}$ quarkonia. Solid lines denote the levels which have been detected experimentally and dashed lines indicate the levels predicted by the potential model.

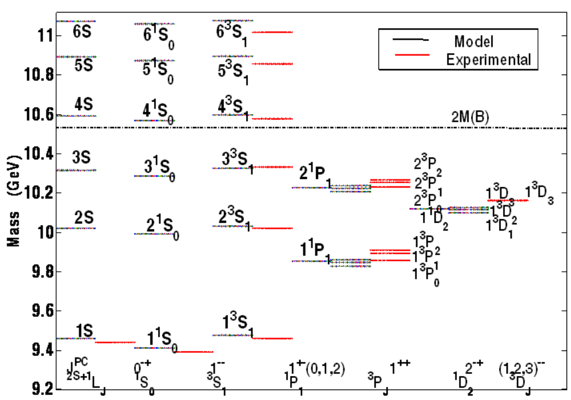

FIG2: Spin dependent mass spectrum of $b\bar{b}$ quarkonia . Solid lines denote the levels which have been detected experimentally and dashed lines indicate the levels predicted by the potential model.

Table1: Ratio of mass splitting χ for L=1, S=1 states of charmonium and bottonium systems.

	$c\bar{c}$		$b\bar{b}$	
	Our work	exp	Our work	exp
1P	.57	.49	.67	.57
2P	.63	-	.70	.67
3P	.67	-	.75	-

In **table 1** we give the values of χ for L=1 and S=1. Slowly increasing value of χ can be attributed to the confinement part of the potential. The calculated values of ΔM_P (difference between center of gravity of ^3P_J anf $^1P_1$ state) is 2.0 Mev against the experimental value 0.9 MeV. Our estimate is closer to the experimental data and compared to those obtained by other authors. The predicted mass of $^1P_1$ state of $b\bar{b}$ system is 9.853 GeV which lies between ^3P_J and $^3P_2$.

The results of our calculations show that the new class of energy dependent potential comprising of confinement plus asymptotic term plus Breit Fermi correction gives a good description of spin dependent mass spectra of $c\bar{c}$ and $b\bar{b}$ systems. The values of ratio of mass splitting χ and ΔM_P are well reproduced.

References :

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