Hyperfine Splitting and Leptonic Decay width

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In the mass spectroscopy of heavy quarkonia, the properties hyperfine splitting and leptonic decay width are directly related to the value of square of the modulus of the wave function at the origin. The study of these quantities can throw light on the short range part of spin spin interaction of the potential. In the present work we have used a new class of energy dependent potential to calculate the mass spectroscopy of charmonium and bottonium in the non-relativistic framework. The potential has the following form.

$$V(r, E_{n,l}) = \frac{1}{2} m\omega^2 r^2 (1+\gamma E_{n,l}) + \frac{g}{r^2} + V_{BF} \quad (1)$$

where

$$V_{BF}(r) = V_{LS}(r)(\vec{L}.\vec{S})$$
$$+V_T(r)\left[S(S+1) - \frac{3(\vec{S}.\vec{r})(\vec{S}.\vec{r})}{r^2}\right]$$
$$+V_{SS}(r)\left[S(S+1) - \frac{3}{2}\right]$$

and μ is the reduced mass If we neglect the Breit- Fermi interaction, Schrodinger equation for this potential offers exact solution .The eigen value equation is

$$E_{n,l} = -\frac{1}{8}a^2\omega^2\gamma + \frac{a\omega}{8}\sqrt{a^2\omega^2\gamma^2 + 16} \quad (2)$$

where

$$a = 4n - 2 + \sqrt{(2l+1)^2 + 8\mu g}$$

The parameters of potential are obtained by fitting the energies of E_{2s}, E_{3S} and E_{1P} with respect to E_{1S}. Once the parameters are fixed, the complete potential with Briet Fermi interaction is used in the Schrödinger equation to calculate mass spectrum of $c\bar{c}$ and $b\bar{b}$. Mass of charm and bottom quark are taken as 1.5 GeV and 5.0 GeV respectively. All calculations have been performed in Mathematica 8.0 by software program obtained by Lucha et al [1] for each quantum state separately. The hyperfine interaction produces splitting between spin triplet and spin singlet states. These values are shown in **table 1** up to 4S state for charmonium and bottonium systems. According to Rayleigh-Schrödinger perturbation theory in the first order the hyperfine splitting is related to the square of the wave function as

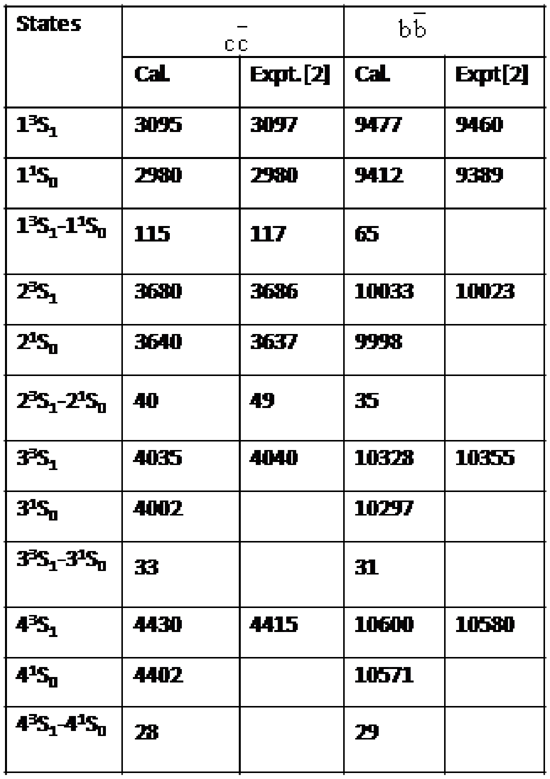
$$\Delta E_{HFS} = \frac{32\pi\alpha_S}{9 m_q m_{\bar{q}}} |\psi(0)|^2 \quad (3)$$

The value calculated from equation (3) is 126 MeV for 1S whereas that obtained from the exact solution is 117 MeV. According to Van Royen formula Leptonic decay width is given by the

$$\Gamma(n^3S_1 \rightarrow e^+e^-) = \frac{16\pi\alpha^2 e_q{}^2}{M_{ns}{}^2 (q\bar{q})} |\psi_{n,s}(0)|^2 \quad (4)$$

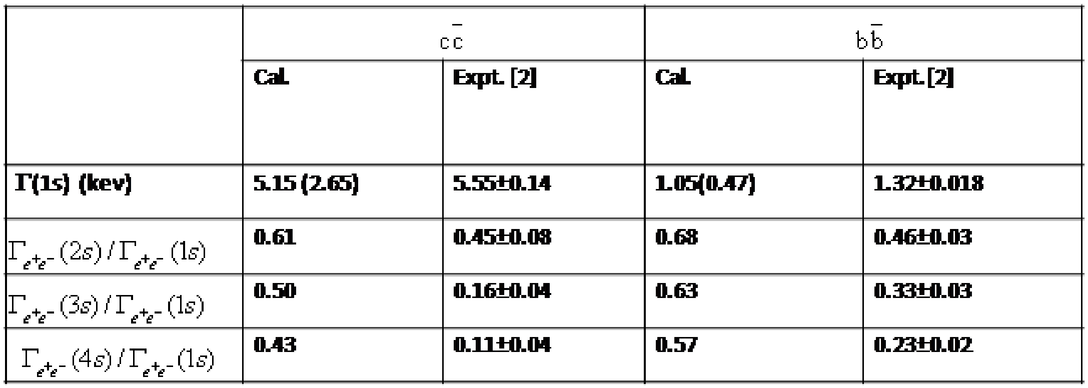
This formula is also true for energy dependent potential. The calculated values of leptonic decay width and its ratio are shown in **table 2 and** compared with experimental data [2]. Also given within brackets are values obtained by Lombard et al [3]. These authors have used the energy dependent potential with only the harmonic oscillator term. Our values are much closer to the experimental values. This shows the importance of asymptotic term in the quark potential in determining the wave functions at origin.

Table 1: Hyperfine splitting and mass spectra in MeV of $c\bar{c}$ and $b\bar{b}$ systems.



States	$c\bar{c}$		$b\bar{b}$	
	Cal.	Expt.[2]	Cal.	Expt[2]
1^3S_1	3095	3097	9477	9460
1^1S_0	2980	2980	9412	9389
1^3S_1-1^1S_0	115	117	65	
2^3S_1	3680	3686	10033	10023
2^1S_0	3640	3637	9998	
2^3S_1-2^1S_0	40	49	35	
3^3S_1	4035	4040	10328	10355
3^1S_0	4002		10297	
3^3S_1-3^1S_0	33		31	
4^3S_1	4430	4415	10600	10580
4^1S_0	4402		10571	
4^3S_1-4^1S_0	28		29	

Table 2: Ratio of Leptonic decay width of different states with that of 1S state.



	$c\bar{c}$		$b\bar{b}$	
	Cal.	Expt. [2]	Cal.	Expt. [2]
Γ(1s) (kev)	5.15 (2.65)	5.55±0.14	1.05(0.47)	1.32±0.018
$\Gamma_{e^+e^-}(2s)/\Gamma_{e^+e^-}(1s)$	0.61	0.45±0.08	0.68	0.46±0.03
$\Gamma_{e^+e^-}(3s)/\Gamma_{e^+e^-}(1s)$	0.50	0.16±0.04	0.63	0.33±0.03
$\Gamma_{e^+e^-}(4s)/\Gamma_{e^+e^-}(1s)$	0.43	0.11±0.04	0.57	0.23±0.02

References:

1. Schoberl.F.F, Lucha. W,1998, HEP-PH/9811453 v

2. Nakamura K. et al. (Particle Data Group) 2010 J. Phys. G: Nucl. Part. Phys. **37:** 075021.

3. R.J. Lombard, J. Mars and C. Volpe, J. Phys G: Nucl. Part. Phys. **34** (2007)1879-188;