Surface Energy of Mesonic($q\bar{q}$) Flux Tube

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Introduction

The striking parallelism between the conventional superconductivity and QCD vacuum is well known and has played pivotal role in explaining the quark confinement mechanism and other closely related issues [1, 2]. The conventional superconductivity predicts the Meissner effect for the magnetic field while the dual supeconductor models leads to Meissner-like effect for the electric field (i.e. dual Meissner effect). In such dual QCD models, the monopoles and/or dyons appear as essential ingredients along with the dual gauge potential in the Lagrabgian and are responsible for the confining features of QCD vacuum with flux tube structure of mesons ($q\bar{q}$) [3]. In this work, we attempt to calculate the surface energy of a mesonic flux tube by using a dual QCD Lagrangain with dyons.

Lagrangian and Field Equations

The Lagrangian density that we consider is derived by using the Zwanziger's two-potential formulation and is given as follows [3]

$$\mathcal{L} = -\frac{1}{4}\tilde{C}^2_{\mu\nu} + |D_\mu\phi|^2 - \lambda(|\phi|^2 - \phi_0^2)^2 \quad (1)$$

where $\tilde{C}_{\mu\nu} = \partial_\mu\tilde{C}_\nu - \partial_\nu\tilde{C}_\mu$ is the dual field tensor and $D_\mu = \partial_\mu - \iota Q\tilde{C}_\mu$ is the covariant derivative. The charge Q appearing here is dyonic and is defined as $Q = \sqrt{e^2 + g^2}$.
The field equations that follow from the Lagrangian density (Eq. 1) are

$$\partial_\mu\tilde{C}^{\mu\nu} + 2Q^2\tilde{C}^\nu|\phi|^2 + \iota QJ^\nu = 0 \quad (2)$$

$$\partial_\mu\partial^\mu\phi - Q^2\tilde{C}_\mu\tilde{C}^\mu\phi - 2\iota Q\tilde{C}_\mu\partial^\mu\phi$$
$$-\iota Q\phi\partial_\mu\tilde{C}^\mu + 2\lambda\phi(|\phi|^2 - \phi_0^2) = 0 \quad (3)$$

where $J^\nu = \phi^*\partial^\nu\phi - \phi\partial^\nu\phi^*$

Nielsen-Olesen Ansatz and Surface Energy

We consider the static case and use the cylindrically symmetric Nielsen-Olesen ansatz [4] for the fields $\tilde{C}_\mu$ and ϕ.

$$\tilde{C}_0 = \tilde{C}_\rho = \tilde{C}_z = 0,\ \tilde{C}_\theta = \frac{C(\rho)}{\rho} \quad (4)$$

$$\phi = f(\rho)e^{\iota n\theta} \quad (5)$$

Using these ansatz in the field equations (Eqs. 2,3), we have,

$$C^{''} - \frac{C^{'}}{\rho} - 2Q^2Cf^2 + 2nQf^2 = 0 \quad (6)$$

$$f^{''} + \frac{f^{'}}{\rho} - \frac{n^2f}{\rho^2} - \frac{Q^2C^2f}{\rho^2}$$
$$+\frac{2nQCf}{\rho^2} - 2\lambda f(f^2 - \phi_0^2) = 0 \quad (7)$$

We introduce new (scaled) variables r, B(r) and F(r) instead of ρ, C(ρ) and f(ρ) as

$$r = \phi_0 Q\rho, \quad B = \frac{Q}{n}C - 1, \quad F = \frac{f}{\phi_0}$$

Eqs. 6 and 7 in terms of the new variables can now be recast as follows,

$$B^{''} - \frac{B^{'}}{r} - 2F^2B = 0 \quad (8)$$

$$F^{''} + \frac{F^{'}}{r} - \frac{n^2FB^2}{r^2} - \frac{2\lambda}{Q^2}F(F^2 - 1) = 0 \quad (9)$$

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Let us define the Ginzburg-Landau (GL) parameter κ as the ratio of the London penetration depth (λ) and the coherence-length (ξ) which are basically the inverse of the vector and sclar mass modes respectively. For the present formulation, the GL parameter turns out $\kappa = \frac{m_\phi}{m_V} = \sqrt{\frac{2\lambda}{Q^2}})$ where m_ϕ and m_V are the mass of doyn field and dual gauge field respectively. The value of κ determines the behaviour of the QCD vacuum (type-I or type-II). For the particular value $\kappa = 1$, corresponding to the transition phase, Eqs. 8 and 9 can be dervied from the first order (Bogomol'nyi type) equations

$$nB^{'} - r(F^2 - 1) = 0 \qquad (10)$$

$$rF^{'} + nBF = 0 \qquad (11)$$

So far, we could not found the exact analytical solutions for Eqs. 8, 9 and 10, 11.

The surface energy (density) of the flux tube can be expressed by calculating the difference of the free energies for the normal and superconducting phases as [5]

$$S = -\frac{1}{4}\tilde{C}_{\mu\nu}^{2} + |D_\mu\phi|^2 + \lambda(|\phi|^2 - \phi_0^2)^2 - DE_c \qquad (12)$$

where $\mathbf{D} = \nabla \times \tilde{\mathbf{C}}$ is the colour-electric field and $E_c = \sqrt{2\lambda}\phi_0^2$ is the critical value of the color electric field. The surface tension for a tube of radius R is of the form

$$\sigma(R) = \frac{1}{R}\int S(\rho)\rho \; d\rho \qquad (13)$$

At the first hand, this expression indicates that there would occur only thin flux tubes (i.e. of small radius) with high surface tension. However, the exact behaviour can only be reported, once we are able to solve the integration $\int S(\rho)\rho \; d\rho$ exactly.

Summary and Conclusions

Using the Lagrangian for dual QCD with dyons, we have derived the field equations for the Nielsen-Olesen ansatz. The field equations are attempted to solve at the boundary where the transition from type-I to type-II superconducting nature of QCD vacuum takes place (i.e. $\kappa = 1$). No analytic solutions could be found even for the first order equations. The surface energy of the mesonic flux tube is then calculated in view of the boundary conditions for the fields in view of the above mentioned field equations. In the absence of exact analytical solution for the field equations at $\kappa = 1$, it is difficult to further simplify the expression for the surface tension of the mesonic flux tube. It still remains to see the evolution of such flux tube structure in view of the change in surface energy and we intend to report on such issue in detail in future.

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