Confinement and Multi-flux Vortices in Dual QCD

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Introduction

The screening effects in a chromodynamic vacuum which acts as a dual superconductor in the background of the magnetic condensation play crucial role in explaining the mechanism of quark confinement [1]. Recently, the colour charge and colour electric flux screening mechanism is investigated in detail [1, 2] and these screening effects are shown to be responsible for the quark (colour) confinement in dual QCD. In this work, it is shown that in addition to such screening effects, there exist n-vortex solutions with Bogomol'nyi-Prasad-Sommerfeld (BPS) conditions with the transition from the type-II to type-I in dual QCD vacuum at strong coupling constant $\alpha_s \simeq 0 \cdot 5$ [3].

Formalism and Lagrangian

The model with an effective Abelian field which describes the strong interaction in QCD (in presence of quarks) with the complex scalar monopole field can phenomenologically be given by the Lagrangian [3, 4] of the following form,

$$\pounds = -\frac{1}{4}\,\tilde{G}_{\mu\nu}\,\tilde{G}^{\mu\nu} + \left|(\partial_\mu + i\,\tilde{g}\,\tilde{C}_\mu)\,\phi\,\right|^2$$

$$+\bar{\psi}\,\gamma^\mu(i\,\partial_\mu + \tilde{g}\,\tilde{C}_\mu)\,\psi - m\,\psi\bar{\psi} - V(\phi^*\phi), \quad (1)$$

where $\tilde{G}_{\mu\nu} = \partial_\mu\tilde{C}_\nu - \partial_\nu\tilde{C}_\mu$ is the field strength corresponding to the dual gauge field $\tilde{C}_\mu$, ϕ is the complex scalar field with the effective magnetic charge $\tilde{g} = 4\pi/g$, and ψ is the quark field with m as free-quark mass [3]. Here g is related to the strong coupling constant of QCD defined as $\alpha_s = g^2/4\pi$ [1]. In view of such screening effects as mentioned in the above section, the dynamics of confinement scenario given by the Lagrangian (1) is automatically derivable from the following Lagrangian in the absence of quarks in a equally capable manner [1, 3],

$$\mathcal{L} = -\frac{1}{4}\,\tilde{G}_{\mu\nu}\tilde{G}^{\mu\nu} + |\mathfrak{D}_\mu\,\phi\,|^2 + \Omega\,(\phi\,\phi^* - \eta^2)^2. \quad (2)$$

Here $\mathfrak{D}_\mu \equiv \partial_\mu + i\,\tilde{g}\,\tilde{C}_\mu$. The energy contents corresponding to above Lagrangian alongwith the formation of the multivortices will be discussed in next section.

Vortices and Energy Contents

Let us consider, the cylindrically symmetric monopole field in two dimensions in the broken phase of symmetry. In fact, the vortices are invariant under translations along any fixed axis and therefore they can be viewed as finite energy solutions in two dimensions [3]. The free energy per unit length associated to them in two dimensions with the suppression of the temporal gauge degrees of freedom of the dual gauge field (i.e. $\tilde{C}_0 = 0$) [2] is then derived in the following form,

$$E = \int d^2x\left[\frac{1}{4}\,\tilde{G}_{ij}^2 + |\mathfrak{D}_k\,\phi\,|^2 + \Omega(\phi\,\phi^* - \eta^2)^2\right]. \quad (3)$$

In view of the screening constraints on the Lagrangian (1) and upon symmetry breaking [2], the free energy from equation (1) or (3) then contains a term $\tilde{m}^2\tilde{C}_i^2$. This term represents the Dual Meissner Effect (DME) and guarantees the confinement of quarks in dual QCD vacuum such that a flux tube structure

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emerge between quark and anti-quark where the energy increases with the separation between them [1, 2]. However, the total energy contents can be re-written in terms of the following squared quantities by using Bogomol'nyi's trick [5] as $E = \int d^2x\, \mathcal{E}$ with,

$$\mathcal{E} = \Big[|(\mathfrak{D}_1 + i\mathfrak{D}_2)\phi|^2 + \frac{1}{2}\{[\tilde{G}_{12} + \tilde{g}(\phi\phi^* - \eta^2)]^2 + (2\Omega - \tilde{g}^2)(\phi\phi^* - \eta^2)^2\} + \tilde{g}\eta^2\, \tilde{G}_{12} \Big]. \quad (4)$$

where $\tilde{G}_{12} = \partial_2 \tilde{C}_1 - \partial_1 \tilde{C}_2$. The integrand of the energy functional given by $\mathcal{E}$ is actually restructured in the form given below by using the Stokes theorem with the elimination of some terms along-with the necessary boundary condition for the monopole field at large distances for finite energy configurations.

Further the boundary of the typ-I and type-II superconducting dual QCD vacuum is of particular interest and if we set a condition $2\Omega = \tilde{g}^2$ (i.e. $g^2 = 6\lambda$) then it automatically restricts the GL parameter κ to its unit value (i.e. $6\lambda/g = 1$). The unit value of the GL parameter for the present model itself represents a transition from type-I to type-II QCD vacuum at a coupling $\alpha_s = 0 \cdot 5\,(\lambda = 1)$. The energy contents now read as,

$$E = \int d^2x\, \Big[\, |\, (\mathfrak{D}_1 + i\mathfrak{D}_2)\phi\, |^2 + \frac{1}{2}[\, \tilde{G}_{12} + \tilde{g}(\phi\,\phi^* - \eta^2)\,]^2 \Big] + \tilde{g}\,\eta^2\,\Phi_{\tilde{E}}. \quad (5)$$

With the vanishing of the squared entities in above expression (i.e. Bogomol'nyi case),

$$(\mathfrak{D}_1 + i\mathfrak{D}_2)\,\phi = 0, \quad (6)$$

$$\tilde{G}_{12} + \tilde{g}\,(\phi\,\phi^* - \eta^2)^2 = 0, \quad (7)$$

one can easily obtain the minimum energy of the system which corresponds to the last term in equation (5) as follows by using the quantistaion condition,

$$E_n = 2n\,\pi\,\eta^2. \quad (8)$$

The equation (8) indicates the linear increase in the energy as the amount of flux increases and the vortices may therefore coalesce into macroscopic regions of colour electric field.

Results and Conclusions

The energy for n-vortex configurations is obtained at the boundary of the type-I and type-II superconducting zones of the present dual QCD vacuum as given by equation (8). It indicates that at the transition point $\alpha_s \simeq 0 \cdot 5$, the quark system belongs to the multi-vortex solutions having a Bogomol'ny's bound on the energy contents. Thus the multi-vortices essentially exists at the border of two types of superconducting regimes of QCD vacuum, and they lost their individuality with the transition from one to other superconducting regime. However, it still remains to see the evolution of the interaction energy among vortices for different couplings/GL parameter (i.e. in the near and far Bogomol'nyi's regime) in both the superconducting zones of dual QCD to investigate the exact process that how the vortices attract/repel each other.

Acknowledgments

One of the authors (HN) would like to thank Prof. T. Lee for some useful suggestions during an early stage of this work. We are grateful to the people of India for their generous support for research.

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