Next to leading order calculation of neutrino mean free path in degenerate quark matter

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Introduction

In this work we calculate the neutrino mean free path (MFP) in dense quark matter. The MFP is related to the neutrino emissivity, which, in turn, is responsible for the cooling of neutron stars. One such calculation was first done by Iwamoto[1]where free fermi gas model was implicitly assumed. In recent years, however, this problem has been revisited by several authors where the correction to the quark dispersion relations in dense system has been incorporated. For example, in [2] the authors have calculated the neutrino emissivity in low temperature quark matter with such corrections by evaluating the quark self-energy in dense system which is referred as non-Fermi liquid (NFL) corrections to the neutrino emissivity.In [3], on the other hand, similar corrections for the MFP was calculated. All these calculations were limited to the leading logrithmic order (LLO) which we extend here to go beyond LLO. The next to leading order corrections show some interesting qualitative behaviour as we shall see below.

Formalism

The MFP is determined by the quark neutrino interaction in dense quark matter *via.* weak processes. We consider the simplest β decay reactions; the absorption process and its inverse,

$$d + \nu_e \to u + e^- \tag{1}$$
$$u + e^- \to d + \nu_e \tag{2}$$

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The neutrino MFP is related to the total interaction rate due to neutrino emission averaged over the initial quark spins and summed over the final state phase space and spins. It is given by[1],

$$\frac{1}{l^{abs}_{mean}(E_\nu, T)} = \frac{g}{2E_\nu}\int \frac{d^3p_d}{(2\pi)^3}\frac{1}{2E_d}\int \frac{d^3p_u}{(2\pi)^3} \frac{1}{2E_u}\int \frac{d^3p_e}{(2\pi)^3}\frac{1}{2E_e}(2\pi)^4\delta^4(P_d + P_\nu - P_u - P_e)|M|^2\{n(p_d)[1-n(p_u)][1-n(p_e)] - n(p_u)n(p_e)[1-n(p_d)]\}, \tag{3}$$

where, g is the spin and color degeneracy, considered to be 6. $|M|^2$ is the squared invariant amplitude is given by $|M|^2 = 64G^2\cos^2\theta_c(P_d\cdot P_\nu)(P_u\cdot P_e)$. Here, we work with the two flavor system as the interaction involving strange quark is Cabibbo suppressed. Interactions within the medium severely modify the onshell self-energy of the quarks which is manifested in the slope of the dispersion relation for the relativistic degenerate plasma. For quasiparticles with momenta close to the Fermi momentum $p_f(i)$, the one-loop self-energy is dominated by the soft gluon exchanges[4]. The quasiparticle energy ω satisfies the relation $\omega = E_p(\omega) + \mathrm{Re}\Sigma(\omega, p(\omega))$ where $E_p(\omega)$ is the single particle energy[5]. The one loop quark self-energy $\Sigma(\omega, p(\omega))$ is dominated by a diagram with a soft gluon in the loop. The analytical expression for the one-loop quark self energy keeping terms beyond leading order, exhibits a logarithmic singularity close to the Fermi surface. Thus the long ranged character of the magnetic interactions spoils the normal Fermi-liquid behavior[6].

Trapped neutrino matter

We now consider the case of degenerate neutrinos *i.e.* when $\mu_\nu \gg T$. So in this case both the direct Eq.(1) and inverse Eq.(2) processes can occur. Consequently, the β equilibrium condition becomes $\mu_d+\mu_\nu = \mu_u+\mu_e$. Neglecting the quark-quark interactions the mean free path can be determined for two conditions. For $|p_f(u) - p_f(e)| \geq |p_f(d) - p_f(\nu)|$

$$\frac{1}{l_{mean}^{abs,D}} = \frac{4}{\pi^3} G^2 \cos^2\theta_c \frac{\mu_u^2 \mu_e^3}{\mu_\nu^2}\Big[1+\frac{1}{2}\Big(\frac{\mu_e}{\mu_u}\Big) + \frac{1}{10}\Big(\frac{\mu_e}{\mu_u}\Big)^2\Big][(E_\nu - \mu_\nu)^2 + \pi^2T^2]\Xi^2 \quad (4)$$

where Ξ is caculated as,

$$\Xi = \Big[1 + g^2 C_f\Big(\frac{1}{12\pi^2} ln\Big(\frac{m}{T}\Big) + \frac{2^{1/3}}{9\sqrt{3}\pi^{7/3}}\Big(\frac{T}{m}\Big)^{2/3} - \frac{140\, 2^{2/3}}{189\sqrt{3}\pi^{11/3}}\Big(\frac{T}{m}\Big)^{4/3} + \frac{6144 - 256\pi^2 + 36\pi^4 - 9\pi^6}{864\pi^6} \times \Big[1 - 3ln\Big(\frac{0.928m}{T}\Big)\Big]\Big(\frac{T}{m}\Big)^2\Big)\Big] \quad (5)$$

Similarly, for $|p_f(d) - p_f(\nu)| \geq |p_f(u) - p_f(e)|$, the corresponding expression for mean free path can be obtained by replacing $\mu_u \leftrightarrow \mu_d$ and $\mu_e \leftrightarrow \mu_\nu$ in Eq.(4). Since quark and electron are assumed to be massless, the chemical equilibrium condition gives $p_f(u) + p_f(e) = p_f(d) + p_f(\nu)$, which we use to derive Eq.(4).

In case of quark-neutrino scattering, we obtain[1],

$$\frac{1}{l_{mean}^{scatt,D}} = \frac{3}{16} n_{q_i}\sigma_0 \times \Big[\frac{(E_\nu - \mu_\nu)^2 + \pi^2T^2}{m_{q_i}^2}\Big]\Xi^2\Lambda(x_i) \quad (6)$$

with n_{q_i} as number density of quark, m_{q_i} as mass of quark, σ_0 a constant and $\Lambda(x_i)$ as defined in[3].

Untrapped neutrino matter

We also derive MFP for nondegenerate neutrinos *i.e.* when $\mu_\nu \ll T$. For nondegenerate neutrinos the inverse process (2) is dropped. Hence, we neglect the second term in the curly braces of Eq.(3). The MFP at next to leading order in T/μ is given by,

$$\frac{1}{l_{mean}^{abs,ND}} = \frac{3C_F\alpha_s}{\pi^4} G^2 \cos^2\theta_c\ \mu_d\ \mu_u\ \mu_e \times \frac{(E_\nu^2 + \pi^2T^2)}{(1 + e^{-\beta E_\nu})}\Xi^2 \quad (7)$$

Similarly, for the scattering of nondegenerate neutrinos in quark matter [1] with appropriate phase space corrections we obtain,

$$\frac{1}{l_{mean}^{scatt,ND}} = \frac{C_{V_i}^2 + C_{A_i}^2}{20} n_{q_i}\sigma_0 \times \Big(\frac{E_\nu}{m_{q_i}}\Big)^2\Big(\frac{E_\nu}{\mu_i}\Big)\Xi^2. \quad (8)$$

Here, we have assumed $m_{q_i}/p_{f_i} \ll 1$.

Conclusion

We have shown that the neutrino MFP receives significant contribution from higher order terms in addition to the logarithmic corrections. We have incorporated results beyond leading logarithmic order to include "plasma"or "quasiparticle"effects which are anomalous (NFL) effects entering through phase space modification. The presence of the logarithmic terms and fractional powers considerably reduce the MFP of the neutrinos that is expected to influence the cooling of compact stars.

References

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