

Energy change due to fluctuations in two-stream QCD plasma

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We calculate the change in energy of a fast moving partons due to chromodynamic field fluctuations in a two-stream plasma. The change in energy grows exponentially with time due to the unstable modes. A strong direction dependence of the energy loss has also been found.

1. Introduction

Relativistic heavy ion collider (RHIC) and Large Hadron Collider (LHC) are designed to produce quark gluon plasma and study its properties at high temperature and/or high density. The possibility of QGP formation at RHIC experiment, with initial density of $5 \text{ GeV}/fm^3$ is supported by the observation of high p_T hadron suppression. This phenomena commonly known as jet jet quenching actually is related to the energy loss of fast moving pal-sama [1].

The estimation of energy loss in quark gluon plasma is thus essential to understand high p_T hadron production in relativistic plasma. The first such estimation was performed by Bjorken [1] long back. Since then, lot of progress has been made to perfect such estimates under various circumstances including both the collisional and radiative losses. Most of these calculations are performed in situations where the distributions of soft partons providing the thermal background are assumed to have isotropic momentum distribution. In realistic scenario, due to rapid initial expansion the system in the longitudinal direction cools faster than the transverse direction leading to $\langle p_L^2 \rangle \ll \langle p_T^2 \rangle$. Such momentum anisotropy might lead to collective modes having characteristic behavior distinct from what happens in isotropic plasma.

Few modes might be unstable. In Ref. [4] it has been shown that a field fluctuation might lead to an energy gain of a moving quark in isotropic QGP irrespective of its velocity. It would therefore be interesting to see what happens in situations where the ground state is anisotropic and the random behavior of the chromodynamic fields are considered.

2. Fluctuation spectrum of two stream palasma

We calculate the energy change of partons due to chromoelectric field fluctuations, propagating in a plasma which is unstable. Instead of considering momentum distribution corresponding to a QGP likely to be produced in relativistic heavy ion collisions we concentrate on a two- stream plasma for the sake of tractibility of our calculations. The distribution function of two stream plasma is

$$f(\mathbf{p}) = (2\pi)^3 n [\delta^{(3)}(\mathbf{p} - \mathbf{q}) + \delta^{(3)}(\mathbf{p} + \mathbf{q})] \quad (1)$$

The two stream plasma is unstable with respect to both electric and magnetic interactions. The effect of these unstable modes in a two stream plasma has been studied recently [5–8].

When a charge particle moves through a plasma it loses part of its energy due to its interaction with medium particles [9].The energy loss of the moving particle per unit time

$$\frac{dE}{dt} = Q^a \mathbf{v} \cdot \mathbf{E}^a |_{\mathbf{r}=\mathbf{v}t}. \quad (2)$$

In presence of field fluctuations and also change of velocity of the particle, Eq.(2) is

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modified and can be written as

$$\frac{dE}{dt} = \langle Q^a \mathbf{v}(t) \cdot \mathbf{E}_t^a(\mathbf{r}(t), t) \rangle, \quad (3)$$

We choose a time interval Δt sufficiently large compared with the period of the random fluctuation of the particle field in the plasma but small compared to the time during which the motion of the particle changes appreciably. During this time interval Δt the particle trajectory differs little from straight line. After solving the classical equation of motion of the particles in the electromagnetic field, we find,

$$\begin{aligned} \frac{dE}{dt} &= \langle Q^a \mathbf{v}_0 \cdot \mathbf{E}^a(\mathbf{r}_0(t), t) \rangle_\beta \\ &+ \frac{Q^a Q^b}{E_0} \int_0^t dt_1 \langle \mathbf{E}^b(\mathbf{r}_0(t_1), t_1) \cdot \mathbf{E}^a(\mathbf{r}_0(t), t) \rangle_\beta \\ &\quad + \frac{Q^a Q^b}{E_0} \int_0^t dt_1 \int_0^{t_1} dt_2 \\ &\left\langle \sum_j E_{t,j}^b(\mathbf{r}_0(t_2), t_2) \times \frac{\partial}{\partial r_{0j}} \mathbf{v}_0 \cdot \mathbf{E}^a(\mathbf{r}_0(t), t) \right\rangle_\beta \end{aligned} \quad (4)$$

We concentrate only on the last two terms which come from velocity and field fluctuations.

Now the dispersion relation $\epsilon_L(\omega, \mathbf{k}) = 0$ gives four roots $\pm\omega_\pm(\mathbf{k})$ for two stream plasma. The dielectric function reads as [6]

$$\begin{aligned} \epsilon_L(\omega, \mathbf{k}) &= \frac{(\omega - \omega_+(\mathbf{k}))(\omega + \omega_+(\mathbf{k}))}{(\omega^2 - (\mathbf{k} \cdot \mathbf{u})^2)^2} \\ &\quad \times (\omega - \omega_-(\mathbf{k}))(\omega + \omega_-(\mathbf{k})), \end{aligned} \quad (5)$$

where, $0 < \omega_+(\mathbf{k}) \in R$ for any value of \mathbf{k} but $\omega_-(\mathbf{k})$ is imaginary for $\mathbf{k}^2(\mathbf{k} \cdot \mathbf{u})^2 < 2\mu^2(\mathbf{k}^2 - (\mathbf{k} \cdot \mathbf{u})^2)$ when it represents the two-stream electrostatic instability generated due to the mechanism analogous to the Landau damping. For $\mathbf{k}^2(\mathbf{k} \cdot \mathbf{u})^2 \geq 2\mu^2(\mathbf{k}^2 - (\mathbf{k} \cdot \mathbf{u})^2)$, the mode is stable, $0 < \omega_-(\mathbf{k}) \in R$. Let us now consider the domain of wave vectors obeying $\mathbf{k}^2(\mathbf{k} \cdot \mathbf{u})^2 < 2\mu^2(\mathbf{k}^2 - (\mathbf{k} \cdot \mathbf{u})^2)$ when $\omega_-(\mathbf{k})$ is imaginary and it represents the unstable electrostatic mode. In this case we write down $\omega_-(\mathbf{k})$ as $i\gamma_k$ with $0 < \gamma_k \in R$ Considering

the correlations between the electric field fluctuations and between the velocity and field fluctuations coming from the contributions of the unstable mode (which are fastest growing function of $(t_1 + t_2)$ and $(t_1 - t_2)$) the second and the third terms in Eq.(4) can be combined to obtain [10]

$$\begin{aligned} \frac{dE}{dt} &= \frac{Q^a Q^b}{E_0} \frac{g^2}{4} \delta^{ab} n \int \frac{d^3 k}{(2\pi)^3} \frac{1}{\mathbf{k}^2} \frac{e^{2t\gamma_k}}{(\omega_+^2 - \omega_-^2)^2} \\ &\quad \times \frac{(\gamma_k^2 + (\mathbf{k} \cdot \mathbf{u})^2)^3 (\gamma_k^2 - (\mathbf{k} \cdot \mathbf{v}_0)^2)}{\gamma_k (\gamma_k^2 + (\mathbf{k} \cdot \mathbf{v}_0)^2)^2} \end{aligned} \quad (6)$$

As can be seen from Eq.(6) the expression for the change in energy diverges. However, for non-zero γ_k the effect of the growth of unstable modes in a two-stream plasma is clearly revealed. This contribution to the energy change due to presence of unstable modes in an expanding plasma should be added to the usual polarization loss.

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