

Simple analytical method to design electrostatic einzel lens

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Introduction

Several types of focusing elements are used in transporting a low energy ion beam (LEIB). Among these most prevalent focusing elements are magneto-static (MS) and electro-static (ES) quadrupoles, which have strong focusing power. These beam elements are somewhat difficult to construct and install accurately for symmetry. Other types of focusing elements are cylindrically symmetric lens, so easy to construct and install in a beam line. They are solenoid, MS Glaser solenoid, MS einzel and ES einzel lenses. One problem associated with the ES lenses is that they should be operated in good vacuum to reduce the probability of electric discharge and break down. Earlier, people used some numerical computer program, which take the geometry of the electrodes in rectangular or cylindrical coordinate for computing electric potential and field distribution. Particle distribution in the starting beam is defined to track them and visualize their movement when projected paraxially with some initial energy from a far end. This gives information about the optical properties of the electrodes constituting the lens. The calculated electrostatic values are used to estimate its optical properties.

Computation of Potential and Field

There are several analytical potential form to find the ES potential and field distribution inside the einzel lens. But here only one closed potential form is discussed. A geometry is assumed consisting of three coaxial metal cylinder of same diameter, D . The gap between the end and centre cylinders is assumed to be S . The potential charged to the end and centre cylinders are V_1 and V_2 . The schematic diagram of the einzel lens and potential charged is depicted in Fig. 1. The potential and field distribution in a gap in the central axis is given by eqs. (1) and (2) respectively taking the origin at the gap-centre.

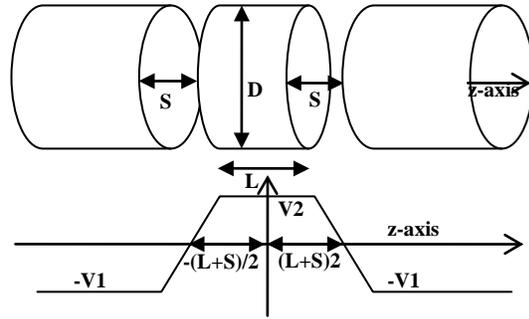


Fig. 1 The schematic sketch of an ES einzel lens and the potential charged on the metal cylinders.

$$\varphi(z) = \frac{(V_2 - V_1)D}{2\pi S} \left\{ z_+ \cdot \tan^{-1}(z_+) - z_- \cdot \tan^{-1}(z_-) \right\} + (V_1 + V_2)/2 \quad (1)$$

$$z_+ = (2z + S)/D, \quad z_- = (2z - S)/D$$

Where $\varphi(z)$ is a monotonic function of z and goes to ± 1 as z approaches $\pm\infty$. It is deduced from electric scalar potential form given by Szilagyí [1] for two apertures at the two ends. This uses a superposition of the potentials for two thin apertures with equal diameter separated by a gap (S).

$$\xi(z) = \frac{-(V_2 - V_1)}{\pi S} (P - Q) \quad (2)$$

$$P = \frac{z_+}{1 + z_+^2} + \tan^{-1}(z_+) \quad (2a)$$

$$Q = \frac{z_-}{1 + z_-^2} + \tan^{-1}(z_-) \quad (2b)$$

When the origin is at the centre of the mid-cylinder and the gap centres are shifted at $d = \pm(L+S)/2$, the total potential, $\varphi_i(z)$ and field, $\xi_i(z)$ distribution on the central axis is given by eqs. (3) and (4) respectively. The potentials and field are plotted in Fig. 2a and 2b respectively for $S=D=10$ cm, $L=20$ cm and $V_1=0$, $V_2=5$ kV, calculated analytically using eq. (3) and (4)

together with the numerical values computed by the POISSON code [2].

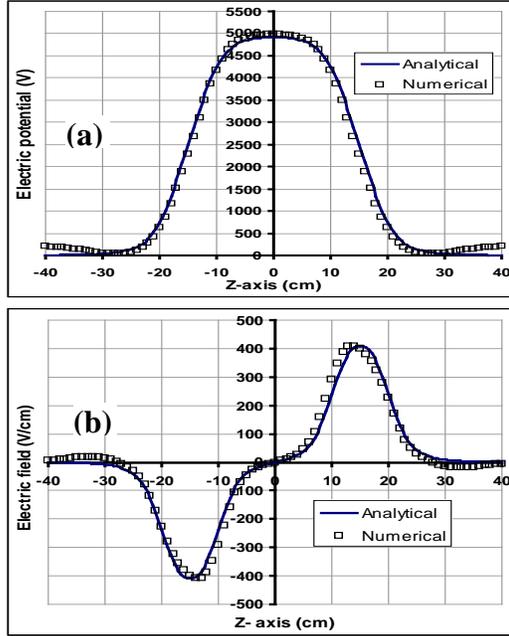


Fig. 2 Plot of electric potential (a) and field (b).

$$\varphi_i(z) = -\varphi(z-d) + \varphi(z+d) + V1 \quad (3)$$

$$\zeta'_i(z) = -\zeta'(z-d) + \zeta'(z+d) \quad (4)$$

Beam Optical Properties

It is very important to find the optical values like focusing power, principal planes etc by simple method. Charged particles projected para-axially get radial kick $qE_r(z)$ inward or outward as $\varphi''(z) > 0$ or < 0 respectively. Particles in motion are properly described by eq. (5) where $T(z) = (\sqrt{3\varphi'(z)}) / (4(V_0 - \varphi(z)))$ and $R = r\varphi^{1/4}(z)$, which is called the reduced ray and V_0 is the initial potential corresponding to the particle energy. If evaluation is done for large number of sectionized weak lenses, for which the reduced ray individually matches with the actual ray. The solution of the eq. (5) is given in matrix eq. (6). The total focal length due to all the thin lenses including drift spaces of width 'd' in between the thin lenses gives accurate particle tracking in the thick lens. Focal length and position of the principal plane for the i-th thin lens are written as in eqs. (7) and (8) respectively. Let $V_0 = 10\text{kV}$ be the potential to attain the initial kinetic energy by

the particle (proton here) and it passes through the einzel lens with potential $V1=0$ and $V2=+5\text{kV}$. The focal length $1/F \approx \sum 1/f_i$, for the whole lens is calculated to be 105.5 cm approximately if the thickness of the individual $i=866$ thin lens be $d=0.1\text{cm}$ and finding $(\varphi(z_i)/\varphi(z_{i-1}))^{1/4} = 1$ for thin lenses is satisfied.

$$R'' + T^2(z)R = 0 \quad (5)$$

$$\begin{pmatrix} R_f \\ R'_f \end{pmatrix} = \begin{pmatrix} \cos(Td) & \frac{\sin(Td)}{T} \\ -T \sin(Td) & \cos(Td) \end{pmatrix} \begin{pmatrix} R_i \\ R'_i \end{pmatrix} \quad (6)$$

$$f_i = 1/(T_i \sin(T_i d)) = 1/(T_i^2 d) \quad (7)$$

$$z_{pi} = (\cos(T_i d) + 1)/(T_i \sin(T_i d)) \quad (8)$$

Adopting the procedure of reference [3] and using the difference formula for general focal length $1/f_i = (\zeta'(z_i) - \zeta'(z_{i-1})) / (4V_0(V_0 - \varphi(z_i)))$ of thin lens and integrating by matrix method, the particles are tracked along the axis and the focal length $F = 110.2\text{ cm}$ is obtained.

Feeding the same geometrical and electrical data, the principal and mid focal lengths were found to be 87.3 cm and 97.6 cm respectively by SIMION code [4]. These are close to the analytical values calculated above, hence corroborating the described simple method of einzel lens design and beam optical evaluation.

Conclusions

The potential and field are evaluated by the simple analytical method for a particular cylindrical einzel lens and optical properties are also evaluated approximately by the simple optical method following the section model. The method design and optical evaluation are supported by the standard code SIMION. The method turns into a tool to design an ES einzel lens for wide applications.

References

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