Chirality in nuclei: where do we stand today?

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The chirality in nuclear rotation was proposed by S. Frauendorf and J. Meng in 1997. Since then, a lot of effort, both from the experimental and theoretical side, has been devoted to explore and understand this rare exotic nuclear phenomenon. Over the years, chiral bands were proposed in several nuclei, mainly, in A∼100 and A∼130 mass regions. But, while looking for more observable experimental fingerprints other than just the energy spectra, the chiral interpretation of bands became doubtful. With the pouring in of more experimental data on absolute $B(M1)$ and $B(E2)$, and innovative theoretical calculations, the chiral character of bands in a few nuclei have been firmly established, and even the transition from chiral vibration to static chirality has been observed. The discovery, progress and recent updates in nuclear chirality, which continues to be a subject of intense discussion, have been reviewed.

1. Introduction

Chirality, an important symmetry in many physical systems, was not either considered and/or visualized in the context of nuclear structure to begin with. Then, not too long ago, it was pointed out that the rotational motion of a triaxial nucleus attains chirality if the rotational angular momentum has substantial projections on all the three principal axes of its triaxial density distribution [1, 2]. The short, long and intermidiate axes form a screw with respect to the total angular momentum vector, which manifests in to either a left-handed or a right-handed system. To visualize the simplest configuration of a nucleus to attain chirality, a nucleus was considered with a valence proton in an orbital just above the Fermi surface (particle), and a valence neutron just below the Fermi surface (hole) [2]. As the core-particle intercation is attractive, to achieve minimal energy, the orientation would be favorable where maximum overlap happens between the proton orbital and the triaxial density. Similarly, because the core-hole interaction is repulsive, the orientation which minimizes the overlap of neutron orbital with the triaxial density would be favorable. These result in the proton aligns its angular momentum along the short axis, and the neutron along the long axis. The angular momentum of the core happens to be of collective nature. This gets oriented along the intermidiate axis with the largest moment of inertia, because the density distribution deviates strongest from the rotational symmetry with respect to this axis. Reverting the direction of the component of the angular momentum of the intermidiate axis changes the chirality. The left-handed and the right-handed configuration, which have the same energy, manifests themselves as two degenerate rotational bands – the chiral doublet.

Since the prediction of nuclear chirality and the first proposed candidate (^{134}Pr) for chiral doubling with the $\pi h_{11/2} \otimes \nu h_{11/2}$ configuration [3], a lot of effort has been devoted to search for this phenomenon. Till date, based on energy spectra, candidate chiral doublet bands have been proposed in a number of odd-odd nuclei in the A∼130 region with the suggested configuration $\pi h_{11/2} \otimes \nu h_{11/2}$, A∼100 region with $\pi g^{-1}_{9/2} \otimes \nu h_{11/2}$, and A∼190 region with $\pi h_{9/2} \otimes \nu i^{-1}_{13/2}$. Apart from these, a few more cases were proposed where more than one valence particle and hole were involved [4–6]. Very recently, the first evidence for chirality in A∼80 region has been reported [7] with the $\pi g_{9/2} \otimes \nu g_{9/2}$

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configuration.

2. Recent experimental developments

With more and more chiral bands getting proposed based on the energy spectra, it has become imperative to look for more experimental signatures. The observed near degeneracy of the two bands is a primary indication of chiral geometry, which can be pinned down in a more definitive way if electromagnetic transition probabilities expected for the twin bands are measured and confirmed experimentally. Selection rules for inter-band and intra-band E2 and M1 transitions were formulated [8, 9]. Most importantly, a measurement of in-band E2 transition rates, which are directly related to the deformation of the bands, has become an essential probe to confirm nuclear chirality.

Only a few lifetime measurements of the proposed chiral bands have been published [10–15] so far. The nearly degenerate pair of bands in ^{134}Pr [3] were indeed considered to be the best example for almost a decade. But, when the electromagnetic transition probabilities for the two bands were measured, it was found that the intraband $B(E2)$ values differ by about a factor of two [10]. This observation was interpreted as suggesting that the change in orientation of the angular momentum vector must be accompanied by a change of shape [10], or even that the results rendered the chiral interpretation itself doubtful [16]. In case of ^{128}Cs, it was established that the electromagnetic transition probabilities are similar [11], but the excitation energies of the two partner bands never approach each other. ^{135}Nd presented a case where the nonplanar geometry is generated by three excited quasiparicles [4] (instead of two, as in the case of ^{134}Pr and ^{128}Cs). For the first time, the splitting between the chiral partner bands was calculated in a microscopic way by extending the tilted-axis-cranking model (TAC) by the random phase approximation (RPA) [12]. The observed transition probablities and energies were found to be in very good agreement with the calculation. It was also shown that the bands are associated with a transition from chiral vibration to static chirality with increasing spin [12]. In the A$\sim$100 mass region, lifetimes of a few levels in the yrast partner of the chiral doublet bands in ^{103,104}Rh were measured [14]. The behaviour of the $B(E2)$ and $B(M1)$ values in both the nuclei were found to be similar. Along with that, the $B(E2)$ values were found to exhibit an odd-even spin dependence, whereas the gradual decreasing trend of $B(M1)$ values were observed with increasing spin [14]. This result has been found to be unique in the sense that the staggering observed in $B(M1)/B(E2)$ ratios is caused by the $B(E2)$ values [14]. It should be noted that in the A$\sim$130 region, the $B(M1)/B(E2)$ staggerings in the odd-odd ^{128}Cs and the odd-A ^{135}Nd were caused by the $B(M1)$ values [11, 12]. In a very recent lifetime measurement of chiral bands in ^{126}Cs, the first observation of the full set of gamma selection rules predicted for strong chiral symmetry breaking has been reported [15]. It has been observed that the electromagnetic properties of both partner bands in that nucleus are similar. Although the interband $B(M1)$ staggering (side $\rightarrow$ yrast) and that too opposite in phase to that of the inband $B(M1)$ staggering was observed earlier in case of ^{135}Nd [12], yrast $\rightarrow$ side interband $B(M1)$ staggering was depicted in ^{126}Cs for the first time [15].

In parallel with these crucial lifetime measurements of already proposed chiral bands, efforts to smell chirality in unknown territory of nuclear landscape continue. Very recently, the first-ever candidate for chiral nuclei in the A$\sim$80 mass region has been proposed (based on energy spectra) in the form of ^{80}Br [7]. This mass region was predicted long back to show chirality [17], but, before this recent work all experimental efforts bore no fruits in that direction. The proposed chiral bands with $\pi g_{9/2} \otimes \nu g_{9/2}$ configuration maintain an energy difference of around $\sim$400 keV within the observed spin interval. The $B(M1)/B(E2)$ ratios for the two bands are comparable in magnitude, with the ratios for band 1 clearly show the odd-even staggering as a function of spin [7]. The almost constant experimental energy staggering parameter ($S(I)$) with a value of $\sim 25\hbar$ is visible clearly for band 1. All these observations propel us to conclude that the case of ^{80}Br might be of chiral vibration [7]. At the same time, the observation of several states with the same spin and parity as of band 2, their closeness in energy with the states of band 2, and their subsequent decay either to the band 1 or band 2 suggest that the chiral geometry of ^{80}Br is not very stable like ^{132}Cs.

3. Theoretical advancements

On the theoretical side, chiral doublet bands were first investigated in the one-particle-one-hole-rotor model (PRM) and the corresponding tilted axis cranking (TAC) approximation for triaxially deformed nuclei [2]. Tilted axis cranking (TAC) [18] is the version of the mean field theory that permits the calculation of the orientation of the deformed field in space together with the parameters that define its shape. Within the TAC mean field approximation, the left-handed and right-handed solutions are exactly degenerate. So, it was not possible to calculate the splitting between the two bands when the nucleus is in chiral vibration regime. This energy difference between the bands is indeed a result of quantum tunnelling between the two solutions. For the first time this energy splitting between the bands in the chiral vibrational regime was demonstrated in ^{135}Nd by extending the tilted axis cranking model by the random phase approximation (RPA) [12]. The RPA calculates the harmonic excitations around the mean field minimum. The RPA solution goes to zero energy, and the tilted mean field assumes a nonzero ϕ value at the transition point to chiral rotation or static chirality. The details about this RPA formalism has been described in the following section.

However, TAC+RPA has its own limitations. Firstly, the angular momentum is not a good quantum number here, and semiclassical approximation is employed to calculate the properties of the electromagnetic transitions. Secondly, the smooth transition from a slow vibration to quantum tunnelling between the left- and right-handed mean field solutions can not be addressed following this approach.

On the other hand, the particle rotor model (PRM) treats the energies and the transition probabilities in a fully quantal manner. The total angular momentum is a good quantum number here. Recently, a particle rotor model has been realized which couples more than one valence protons and neutrons to a rigid triaxial rotor core [19]. Employing this newly developed model, the chirality of the configuration $\pi h^2_{11/2} \otimes \nu h^{-1}_{11/2}$ in the odd-A nucleus ^{135}Nd has been revisited in a fully quantal manner. All the experimental observables, which include the energy spectra of the doublet bands and the reduced transition probabilities $B(M1)$ and $B(E2)$ for intraband as well as for interband have been very well reproduced [19]. After the success of TAC+RPA, this model reaffirms the change from a soft chiral vibration to nearly static chirality in ^{135}Nd [12]. Maximal chirality is reached at spin $I = 39/2$, and it is this point where the two bands come closest even experimentally. In the same work [19], it has been also shown that the nucleus ^{135}Nd resorts back to another type of chiral vibration immediately after this transient phenomenon of static chirality.

Although the particle rotor model has been applied to investigate the nuclear chirality extensively, it is a phenomenological model based on a rigid triaxially deformed core. This may not be that appropriate as the nuclei where chiral doublets have been found are considered to be soft with respect to the triaxiality parameter γ. An alternative approach is based on the interacting boson fermion-fermion model (IBFFM) which takes into account the deformation of the core as an additional degree of freedom [10, 20]. Here, the yrast band is basically built on the ground state configuration of the triaxial core, whereas, the collective structure of the yrare band contains a large component of the γ band and higher-lying collective core structures in the high angular momentum regime. This approach has been quite successful in reproducing the recent experimental observables [10, 20]. The success of this model indicates that shape fluctuations are an essential ingredient for the proper description of the structure of the two bands.

A. Random Phase Approximation

A more advanced quantum mechanical description should also include the quantum fluctuations around the mean field minimum. The Random Phase Approximation (RPA) method makes it possible to include quantum fluctuations in a harmonic order. The quantal fluctuations in a nucleus lead not only to a series of collective excitations like rotations and vibrations but also give rise to correlations in the ground state which changes its properties. They specially lower the ground state energy, thus improving the mean field solution [21] where such correlations are not taken into account.

RPA Formalism

After solving the mean field problem the Hamiltonian can be written as

$$H = h_{mf} + H_{res} \tag{1}$$

where h_{mf} is the diagonal mean field Hamiltonian

$$h_{mf} = E_{mf} + \sum_k e_k \alpha_k^\dagger \alpha_k \tag{2}$$

in terms of the self-consistent mean field quasi particle operators α_k, and H_{res} is the residual interaction. We introduce the quasi-boson approximation $b_\mu^\dagger = \alpha_i^\dagger \alpha_j^\dagger$, where the $b_\mu^\dagger$ are treated as exact bosons and $\mu \equiv i > j$. The Hamiltonian H is rewritten in RPA order by retaining terms up to the second order in the boson operators [22],

$$H_{RPA} = E_{mf} + \sum_{\mu,\nu} A_{\mu,\nu} b_\mu^\dagger b_\nu + \frac{1}{2} \sum_{\mu,\nu} (B_{\mu,\nu} b_\mu^\dagger b_\nu^\dagger + h.c.) \tag{3}$$

where matrices **A** and **B** are hermitian and determined from the residual interaction [22]. The matrix elements for the QQ-interaction are

$$A_{\mu,\nu} = -\delta_{\mu,\nu} E_\mu + \sum_{m=-2}^{2} \kappa q_\mu^m q\nu^{m*} \tag{4}$$

$$B_{\mu,\nu} = \sum_{m=-2}^{2} \kappa q_\mu^m q\nu^{m*} \tag{5}$$

where q_μ^m are the quadrupole matrix elements in quasi-boson approximation [22]

$$Q^m = \sum_\mu q_\mu^m b_\mu^\dagger + q_\mu^{m*} b_\mu \tag{6}$$

$$q_\mu^m = \langle b_\mu Q^m \rangle \tag{7}$$

and $E_\mu = e_i + e_j$ are two quasi-particle energies. The summation over the shell number N is suppressed. We solve the RPA equations

$$\left[H_{RPA}, O_\lambda^\dagger\right] = E_{RPA} O_\lambda^\dagger \tag{8}$$

using the strength function method of Ref. [22] and Ref. [23]. The RPA eigenmode operators $O_\lambda^\dagger$ are

$$O_\lambda^\dagger = \sum_\mu X_\mu^\lambda b_\mu^\dagger - Y_\mu^\lambda b_\mu \tag{9}$$

where the RPA amplitudes X_μ^λ and Y_μ^λ are obtained by solving the standard set of linear equations resulting from Eqn. 8 together with the normalization condition

$$\left[O_\lambda, O_{\lambda'}^\dagger\right] = \sum_\mu X_\mu^\lambda X_\mu^{\lambda'} - Y_\mu^\lambda Y_\mu^{\lambda'} = \delta_{\lambda\lambda'} \tag{10}$$

Since we use a separable force, this set of linear equations is strongly simplified [23].

There are two rotational spurious solutions in the RPA spectrum. One at zero energy induced by the J_z operator and one at the rotational frequency ω induced by the J_+ operator. Numerically the spurious solutions decouple from the physical RPA solutions in a stable manner if the mean field problem is solved accurately enough.

The harmonic vibration around the mean field minimum is realized by RPA. The RPA phonon energy gives the energy splitting between the zero-phonon lower band and the excited one phonon band at a given rotational frequency, ω. While dealing with the chiral phenomenon in nuclei, it describes the system as long as we are in the chiral vibrational regime well before the transition to static chirality. The transition point corresponds to where the RPA solution goes to zero energy and the tilted mean field acquires a nonzero ϕ. Again, from the RPA amplitudes, we can derive the inter-band transition rates using the method of Ref. [23].

4. Multiple chiral doublet bands

The possibility of having more than one pair of chiral doublet bands in a single nucleus has been of recent interest. Following the microscopic relativistic mean field (RMF) approach, the possibilities of multiple chiral doublets (MχD) have been demostrated [24]. The adiabatic and configuration-fixed constrained triaxial RMF approaches were developed and applied to ^{106}Rh [24]. Indeed, the possible existence of more than one pair of chiral rotational bands in one single nucleus has been suggested in ^{106}Rh based on the triaxial deformations and their corresponding high-j proton-hole, neutron-particle configurations. Similar approaches have been extended to other Rh isotopes, and possible occurrence of MχD have been predicted in ^{104,106,108,110}Rh isotopes as well [25]. All these investigations not only further bolster the prediction of MχD in ^{106}Rh, but also opens up new experimental opportunities to search for MχD in A$\sim$100 and other mass regions.

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