

## Mixing ratios and rotational parameters in $^{177}\text{Lu}$ and $^{177}\text{Hf}$

S. Deepa\*, K. Vijay Sai and K. Venkataramaniah

Department of Physics, Sri Sathya Sai Institute of Higher Learning, Prasanthinilayam 515134, INDIA

\* email: deepaseetharaman@sssihl.edu.in

### Introduction

The decay of  $^{177m}\text{Lu}$ , populating high spin states in  $^{177}\text{Lu}$  and  $^{177}\text{Hf}$  is one of the few cases of radioactive decay, which provide an opportunity to study nuclear properties that are generally observable only in medium or high energy nuclear reactions. From this decay, the g factors ( $g_K$ ,  $g_R$ ) associated with rotational and intrinsic structure of the rotational bands in  $^{177}\text{Lu}$  and  $^{177}\text{Hf}$  were extracted earlier [1-3] using the ratio of  $\gamma$ -ray branching intensities ( $\lambda$ ) of the intraband crossover and cascade transitions of the rotational levels (J). The corresponding relations from the rotational model are:

$$\lambda = \frac{I_{\gamma,\Delta J=2}}{I_{\gamma,\Delta J=1}} \dots (1) \quad \text{and}$$

$$\frac{\delta^2}{1+\delta^2} = \frac{2K^2(2J-1)\lambda}{(J+1)(J-1+K)(J-1-K)} \times \left(\frac{E_{\gamma,\Delta J=1}}{E_{\gamma,\Delta J=2}}\right)^5 \dots (2)$$

where K is the quantum number associated with the rotational band. As these equations are valid only in the strong coupling limit, experimental mixing ratios are needed. Although the mixing ratios and g factors were determined in the  $\gamma$ - $\gamma$  directional correlation study ( $9/2^+$ [624] rotational band only) of Hubel et al. [4] and  $\gamma(\theta, H)$  nuclear orientation study of Krane et al. [5], the mixing ratios were not obtained before from the internal conversion coefficients, as no complete and precise studies were reported on the determination of the latter. This gap was filled in the present study, as the mixing ratios of the intraband M1+E2 transitions in all the 3 rotational bands were obtained from the ICCs. The factor  $(g_K - g_R)/Q_0$  is related to the mixing ratios  $\delta$  and  $E_\gamma$  (in MeV) as:

$$\frac{(g_K - g_R)}{Q_0} = \frac{0.933 \times E_{\gamma,\Delta J=1}}{\delta \sqrt{J^2 - 1}} \dots (3)$$

Equations 1, 2, 3 are from Mullins et al [3].

### Experiment

The radioisotope  $^{177}\text{Lu}$  (free from radionuclide impurities) was produced by

neutron irradiation of enriched  $^{176}\text{Lu}$  at the Bhabha Atomic Research Centre, Mumbai, India and was obtained in the form of  $\text{LuCl}_3$  in HCl. As the ground state of the  $\beta^-$  unstable  $^{177}\text{Lu}$  has  $t_{1/2} = 6.647$  d, the initial activity was that of this short-lived radioisotope. Weak  $\gamma$  rays from the decay of the 160.44 d isomeric state in  $^{177}\text{Lu}$  indicated the excitation of this state in the irradiation. The radioisotope was hence allowed to decay for a period of 6 months, till the short-lived activities had become negligible.

For the conversion electron intensity measurements, a mini-orange electron spectrometer that has a mini orange filter and  $\text{LN}_2$  cooled EG & G ORTEC Si(Li) detector was used [6]. The spectrometer's transmission characteristics were studied with standard sources. The gamma spectra were acquired with the gamma spectroscopy system based on a coaxial EG & G ORTEC HPGGe detector. The unshielded detector's energy calibration and efficiency calibration were performed with IAEA standard sources. The singles gamma spectra were acquired for about  $10^6$  s. Spectra acquisition and analysis was performed with GammaVision-32. The interactive computer program FIT that fits analytical models to spectra was also used. The conversion coefficients were determined by the Normalized Peak to Gamma method, by normalizing the measured conversion electron intensities and gamma intensities with that of the 228 keV E2 transition. The theoretical value [7] of  $\alpha_K$  of the said transition was used for normalization. Table 1 lists the K-conversion coefficient values of the M1+E2 transitions that were determined. Many of these are reported for the first time. They are found to agree well with the theoretical conversion coefficients [7].

### Results

Mixing ratio ( $|\delta|$ ) of the M1+E2 multipole transitions in  $^{177}\text{Lu}$  and  $^{177}\text{Hf}$  given in Table 1 were obtained using the present  $\alpha_{K(\text{exp})}$ . Most of

these agree within uncertainties with those determined in the nuclear orientation study of the decay of  $^{177m}\text{Lu}$  by Krane et al. [5]. Using the present  $|\delta_{\text{exp}}|$  the values of  $|(g_K - g_R)/Q_0|$  of the rotational bands exposed in the decay were determined and are shown in Table 2. There is a good agreement between the values of  $|(g_K - g_R)/Q_0|$  obtained independently from our gamma intensity branching ratios and our internal conversion coefficients. The weighted averages shown in the last two rows of Table 3 prove the consistency of our measurements. Using  $g_R = 0.23(2)$  and  $Q_0 = 7.2(1)$  for the rotational bands in  $^{177}\text{Hf}$  and  $g_R = 0.320(11)$  and  $Q_0 = 7.26(4)$  for that of  $^{177}\text{Lu}$ , the values of  $g_K$  obtained for the three bands are shown in Table 4. These results are in agreement with the  $^{176}\text{Yb}(^9\text{Be}, 4n\gamma)^{177}\text{Hf}$  reaction [3], in which the pure rotational character of the  $K = 7/2^-$  was identified, and evidence for coriolis mixing in the  $K = 9/2^+$  band were given. The  $K = 7/2^+$  rotational band in  $^{177}\text{Lu}$  is known to be of pure rotational character.

**Table 1**

$E_\gamma$ (keV)	Transition	Exptl. $\alpha_K$	$ \delta $
$^{177}\text{Lu}$			
121.64	$9/2^+ \rightarrow 7/2^+$	1.41(9)	0.66(12)
147.15	$11/2^+ \rightarrow 9/2^+$	0.91(9)	0.46(23)
171.87	$13/2^+ \rightarrow 11/2^+$	0.61(7)	0.38(30)
195.51	$15/2^+ \rightarrow 13/2^+$	0.37(6)	0.69(33)
218.09	$17/2^+ \rightarrow 15/2^+$	0.30(2)	0.49(15)
$^{177}\text{Hf}$			
105.37	$11/2^+ \rightarrow 9/2^+$	2.50(17)	0.47(13)
112.95	$9/2^- \rightarrow 7/2^-$	0.83(5)	4.04(24)
128.5	$13/2^+ \rightarrow 12/2^+$	1.46(8)	0.43(12)
136.71	$11/2^- \rightarrow 9/2^-$	0.60(9)	2.38(36)
153.29	$15/2^+ \rightarrow 13/2^+$	0.90(6)	0.40(16)
174.40	$17/2^+ \rightarrow 15/2^+$	0.58(5)	0.56(17)
204.09	$19/2^+ \rightarrow 17/2^+$	0.40(2)	0.41(11)
214.43	$21/2^+ \rightarrow 19/2^+$	0.343(14)	0.47(9)

**References**

[1] P. Alexander, F. Boehm, and E. Kankleit, Phys. Rev. 133, (1964) B 284  
 [2] A. J. Haverfield, F. M. Bernthal, and J. M. Hollander, Nucl. Phys. A94, (1967). 337  
 [3] S. M. Mullins, A. P. Byrne, G. D. Dracoulis, T. R. McGoram, and W. A. Seale, Phys. Rev. C 58, (1998) 831.  
 [4] H. Hubel, C. Gunther, K. Krien, H. Toschinski, K.H. Speidel, B. Klemme, G. Kumbartzki, L. Gidefeldt, E. Bodenstedt, Nucl. Phys. A127, (1969) 609.

**Table 2**

Level	$ (g_K - g_R)/Q_0 ^*$		
	[2]	Present <sup>a</sup>	Present <sup>b</sup>
$^{177}\text{Lu}: 7/2^+[404]$			
9/2 <sup>+</sup>	-	-	0.045(8)
11/2 <sup>+</sup>	0.051(4)	0.0486(17)	0.063(32)
13/2 <sup>+</sup>	0.049(3)	0.0475(7)	0.075(59)
15/2 <sup>+</sup>	0.051(3)	0.0440(11)	0.041(20)
17/2 <sup>+</sup>	0.050(4)	0.0478(7)	0.056(17)
$^{177}\text{Hf}: 7/2^-[514]$			
9/2 <sup>-</sup>	-	-	0.0068(4)
11/2 <sup>-</sup>	0.0066(4)	0.0142(5)	0.0114(17)
13/2 <sup>-</sup>	0.016(4)	0.0122(13)	-
15/2 <sup>-</sup>	0.015(5)	0.0047(6)	-
17/2 <sup>-</sup>	-	0.0035(4)	-
19/2 <sup>-</sup>	-	0.0067(8)	-
21/2 <sup>-</sup>	-	0.0062(12)	-
$^{177}\text{Hf} 9/2^+[624]$			
11/2 <sup>+</sup>	-	-	0.044(12)
13/2 <sup>+</sup>	0.052(3)	0.0535(5)	0.050(14)
15/2 <sup>+</sup>	0.053(3)	0.0523(4)	0.055(22)
17/2 <sup>+</sup>	0.054(3)	0.0507(5)	0.040(12)
19/2 <sup>+</sup>	0.055(3)	0.0510(5)	0.056(15)
21/2 <sup>+</sup>	0.052(3)	0.0498(6)	0.047(9)

\* Sign of mixing ratio adopted from Krane et al [5].

<sup>a</sup>  $|\delta|$  determined from gamma branching intensity.

<sup>b</sup>  $|\delta|$  determined from ICC<sub>exp</sub>.

**Table 3**

Method/ Ref.	Weighted average of $ (g_K - g_R)/Q_0 $		
	$^{177}\text{Lu}$ 7/2 <sup>+</sup> [404]	$^{177}\text{Hf}$ 7/2 <sup>-</sup> [514]	$^{177}\text{Hf}$ 9/2 <sup>+</sup> [624]
$\lambda$ . [2]	0.0502(17)	0.013(2)	0.0532(13)
$\gamma$ - $\gamma$ ( $\theta$ ) [4]	-	-	0.0466(7)
$\gamma$ ( $\theta$ , H) [5]	0.0470(34)	0.0051(2)	0.0585(27)
$\lambda$ [3]	0.0521(22)	0.0022(1)	0.0499(6)
ICC (Pr)	0.047(7)	0.0070(4)	0.047(5)
$\lambda$ (Pr)	0.0471(4)	0.0072(2)	0.0516(2)

**Table 4**

Nucleus/ Band	$g_R$ (present)	$g_R$ [3]
Hf 7/2 <sup>-</sup> [514]	0.23(2)	0.23(2)
Hf 9/2 <sup>+</sup> [624]	-0.11(4)	-0.12(4)
Lu 7/2 <sup>+</sup> [404]	0.66(5)	-

[5] K. S. Krane, C. E. Olsen, and W. A. Steyert, Phys. Rev. C. 10 (1974) 825.  
 [6] S. Deepa, K. Vijay Sai, R. Gowrishankar, Dwarakarani Rao and K. Venkataramaniah Appl. Radiat. and Isot., 69 (2011) 869.  
 [7] I. M. Band, M. B. Trzhaskovskaya, C. W. N. Jr., P. Tikkanen, and S. Raman, At. Data and Nucl. Data Tables 81, (2002) 1.