Effect of Tensor Interaction in Spin-orbit Splitting of Shell Model States

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The genesis of magic numbers is known to be due to a strong spin-orbit interaction in nuclei [1-3]. However, the evolution of single particle energy levels with increasing N or Z forming islands of stability does not follow a simple geometrical rule. With the advent of new experimental facilities like radioactive ion beams it has been observed that new areas of magicity have developed [4-6] while conventional shell gaps have weakened thus necessitating a relook in the mean field type of nuclear structure calculations.

The spin-orbit potential in Skyrme Hartree-Fock theory with the inclusion of tensor component is given by

$$V_{s.o.}^{q} = \frac{W_0}{2r}\left(2\frac{d\rho_q}{dr} + \frac{d\rho_{q'}}{dr}\right) + \left(\alpha\frac{J_q}{r} + \beta\frac{J_{q'}}{r}\right) \quad (1)$$

where $J_{q(q')}$ (r) is the proton or neutron spin-orbit density defined as

$$J_{q(q')}(r) = \frac{1}{4\pi r^3} \sum_i v_i^2(2j+1)\left[j_i(j_i+1) - l_i(l_i+1) - \frac{3}{4}\right] R_i^2(r) \quad (2)$$

The tensor interaction is given by

$$V_T = \frac{T}{2}\left\{\left[(\boldsymbol{\sigma_1}.\boldsymbol{k}')(\boldsymbol{\sigma_2}.\boldsymbol{k}') - \frac{1}{3}(\boldsymbol{\sigma_1}.\boldsymbol{\sigma_2})k'^2\right]\delta(\boldsymbol{r_1}-\boldsymbol{r_2}) + \delta(\boldsymbol{r_1}-\boldsymbol{r_2})\left[(\boldsymbol{\sigma_1}.\boldsymbol{k})(\boldsymbol{\sigma_2}.\boldsymbol{k}) - \frac{1}{3}(\boldsymbol{\sigma_1}.\boldsymbol{\sigma_2})k^2\right]\right\} + U\{(\boldsymbol{\sigma_1}.\boldsymbol{k}')\delta(\boldsymbol{r_1}-\boldsymbol{r_2})(\boldsymbol{\sigma_1}.\boldsymbol{k}) - \frac{1}{3}(\boldsymbol{\sigma_1}.\boldsymbol{\sigma_2}) \times [\boldsymbol{k}'.\delta(r_1-r_2)\boldsymbol{k}]\} \quad (3)$$

The coupling constants T and U denote the strength of the triplet-even and triplet-odd tensor interactions respectively.

We have used SKP set of parameters to calculate the splitting of single particle shell model states of ^{208}Pb where there is abundant experimental data [7]. In Table I the calculated splitting of the spin-orbit partners of proton and neutron single particle states around ^{208}Pb have been presented along with the experimental values. For proton states nuclei ^{207}Tl and ^{209}Bi have been considered for comparison whereas for neutron states nuclei ^{207}Pb and ^{209}Pb were considered. The level splittings are reproduced quite admirably by SKP parameter set with the inclusion of tensor interaction thus vindicating the importance of inclusion of tensor interaction.

TABLE I

Splitting of spin-orbit doublets

Protons	Energy in MeV					
	Sly5	Sly4	SKM*	SIII	SKP-T	EXP
Δ1h	6.43	6.22	5.94	5.20	4.64	5.56
Δ2d	1.95	1.89	2.37	1.63	1.55	1.33
Δ2f	2.69	2.61	2.57	2.32	2.17	1.93
Δ3p	1.05	1.02	0.97	0.87	0.79	0.84
Neutrons						
Δ1h	5.83	5.59	5.55	4.73	5.32	5.10
Δ1i	7.65	7.25	7.26	6.39	6.93	6.46
Δ2f	2.05	1.96	2.93	2.67	2.27	2.03
Δ2g	3.68	3.57	3.58	3.30	2.83	2.51
Δ3p	1.17	1.13	1.13	1.02	0.84	0.90
Δ3d	1.72	1.67	1.60	1.47	1.31	0.97

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