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Prediction of K-Value for Super-deformed Bands from the VMI Model

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I. Introduction

For the super-deformed (SD) bands, gamma ray energies are only the spectroscopic information available till today. Projection of Angular momentum (K-value) along the symmetry axis or the lowest spin (I_0) is unknown in most of the SD bands. Because of the non-observation of the discrete linking transitions between the super deformed states and the low lying states at Normal Deformation (ND), the experimental data for the spin of the rotational bands is poor. One of the most useful information to study the SD bands requires essentially the K value and the band head spin. VMI model for odd-odd nuclei developed by Goel et al is modified to estimate the K value or lowest spin state in super deformed nuclei in terms of gamma energy ratio. The energy ratio is expressed in terms of gamma ray energies as, R= E_γ (I+6 to I+2) / E_γ (I+4 to I+2). The ratio depends on the K and σ value. The softness parameter (σ) was obtained through a two parameter formula by best fit method (BFM). A program was developed and executed using C++. VMI results were compared with Shalaby's two parameter formula. They have determined the lowest spin I_f or K value by drawing the dynamic and static moments of inertia against the rotational frequency for different values of spin. From these figures assigned the critical spin below which the normal behavior of the band is reversed. This critical spin is to regard as the baseline spin or the lowest spin of the superdeformed band. We have compared our results with them and good agreement is obtained between the two.

II. Methodology

The band head energy (E_{IK}) of rotational bands in a VMI model is given as

$$E_{IK} = \frac{1}{2\mathcal{g}_I}\left[I(I+1) - K(K+1)\left\{ 1 + \frac{I(I+1)-K(K+1)}{4C\mathcal{g}_I^3} \right\} \right]$$

Where $\mathcal{g}_I$ is moment of inertia and C is the restoring force constant related to σ.

For energy in state defined by,

$$E_{IK} = \frac{I(I+1)-K(K+1)}{4\mathcal{g}_K}\left[\frac{(3r_{IK}-1)}{r_{IK}^2} \right]$$

Where,

$$r_{IK} = \left\{ \frac{1}{3} + \frac{2}{3}\mathrm{Cosh}\left[\frac{1}{3}\mathrm{Cosh}^{-1}\left\{ 27 \times \frac{1}{2}\left(\frac{\sigma\, I(I+1) - K(K+1)}{(2K+1)} + \frac{2}{27} \right) \right\} \right] \right\}$$

The energy ratio can be written in terms of gamma energies as,

$$R = \frac{E_\gamma(I+6 \to I+2)}{E_\gamma(I+4 \to I+2)}$$

We get,

$$= \frac{(6K+21)\left\{ \frac{\mathrm{Cosh}\left[\frac{1}{3}\mathrm{Cosh}^{-1}\left(\sigma \times 27 \times \frac{6K+21}{2K+1}+1\right)\right]}{\left\{\frac{1}{3}+\frac{2}{3}\mathrm{Cosh}\left[\frac{1}{3}\mathrm{Cosh}^{-1}\left(\sigma \times 27 \times \frac{6K+21}{2K+1}+1\right)\right]\right\}^2} \right\} - (2K+3)\left\{ \frac{\mathrm{Cosh}\left[\frac{1}{3}\mathrm{Cosh}^{-1}\left(\sigma \times 27 \times \frac{2K+3}{2K+1}+1\right)\right]}{\left\{\frac{1}{3}+\frac{2}{3}\mathrm{Cosh}\left[\frac{1}{3}\mathrm{Cosh}^{-1}\left(\sigma \times 27 \times \frac{2K+3}{2K+1}+1\right)\right]\right\}^2} \right\}}{(4K+10)\left\{ \frac{\mathrm{Cosh}\left[\frac{1}{3}\mathrm{Cosh}^{-1}\left(\sigma \times 27 \times \frac{4K+10}{2K+1}+1\right)\right]}{\left\{\frac{1}{3}+\frac{2}{3}\mathrm{Cosh}\left[\frac{1}{3}\mathrm{Cosh}^{-1}\left(\sigma \times 27 \times \frac{4K+10}{2K+1}+1\right)\right]\right\}^2} \right\} - (2K+3)\left\{ \frac{\mathrm{Cosh}\left[\frac{1}{3}\mathrm{Cosh}^{-1}\left(\sigma \times 27 \times \frac{2K+3}{2K+1}+1\right)\right]}{\left\{\frac{1}{3}+\frac{2}{3}\mathrm{Cosh}\left[\frac{1}{3}\mathrm{Cosh}^{-1}\left(\sigma \times 27 \times \frac{2K+3}{2K+1}+1\right)\right]\right\}^2} \right\}}$$

The K value is calculated in terms of σ. The value of σ is obtained by the best fit method and it is given as constant parameter. The results obtained from this equation are compared with Shalaby's results are given in Table 1.

Table 1. Shalaby's results compared with K value by VMI model equation. The softness parameter (σ) is obtained by BFM.

SD Bands	Softness parameter $\sigma \times 10^{-4}$	R value (experimental)	K value from VMI model	K value from Shalaby's table
^{58}Ni (b1)	9.33	2.17	8	11
^{58}Cu	0.206	2.24	5	6
^{59}Cu(b1)	7.36	2.15	9.5	11.5
^{61}Zn	20.33	2.12	13.5	15.5
^{62}Zn	50.17	2.10	16	20
^{65}Zn	2.65	2.13	11.5	18.5
^{68}Zn	1.45	2.10	16	16
^{84}Zr	10.87	2.08	23	23
^{86}Zr	36.46	2.08	23	25
^{89}Tc	0.153	2.09	18.5	21.5

References

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