Prediction of energies of yrast band in some even-even nuclei

A. K. Varshney[1], Yuvraj Singh[2], *M. Singh[3], Chhail Bihari[4],
S. K. Dhiman[5], K.K. Gupta[2] and D.K. Gupta[3]

1. *R.G.M. Govt. P.G. College, Jogindar Nagar -175015 (HP), India*
2. *Govt. College, Dhaliara (HP)-177103, India*
3. *S. S. L. D. Varshney Girls Engg. College, Aligarh - 202001 (UP), India*
4. *Bon Maharaj Engg. College, Vrindavan, Mathura (UP)-281121, India*
5. *H. P. University, Shimla (HP), India*

*** singh.moti@gmail.com**

Introduction:

The deformation parameter β and γ of the collective model of Bohr and Mottelson [1] are basic descriptors of the nuclear equilibrium shape and structure. The researchers found that the values of γ obtained from energies (= γ_e) are nearly equal to the value of γ derived from transition rate (= γ_b) in even Xe, Ba and Ce nuclei (A~120-140) and Hf, W, Os, Pt and Hg nuclei (A~160-200) using rigid triaxial rotor model of Davydov – Filippov [2-3]. In the present study, the relatively light mass nuclei (Mo, Ru and Pd) have been taken. As far as γ is concerned, it is known that the Ru chains of nuclei is intermediate between the two having opposite trends for parameter γ, decreasing for Mo & increasing for Pd, and has an irregular behaviour in itself with the increase of neutron number [4].

Procedure for evaluating asymmetric parameter (γ) and moment of inertia (I_0):

The Hamiltonian in Asymmetric Rotor Model of Davydov – Filippov [5] is written as

$$H = \frac{\hbar^2}{2}\sum \frac{I_i^2}{J_i} \tag{1}$$

Where, I_i is the projection of the angular momentum on the intrinsic axes. The moment of inertia of the model are given by the hydrodynamic relation

$$J_i = \frac{4}{3}J_0 \sin^2(\gamma - \frac{2\pi}{3}) \tag{2}$$

Where, $J_0 = 4B\beta^2$ (3)

Simple analytical expressions for the energy of two levels of the required symmetry are, for $I = 2$, defined by the expressions

$$E_{2\,1,2}^{+} = \frac{9[1-(-1)^{\sigma_{1,2}}\sqrt{1-\frac{8}{9}X}}{X} \tag{4}$$

In the units of $\hbar^2/J_0$ (5)

where $\sigma_{1,2} = 0, 1$

and $X = \sin^2 3\gamma$

(a) Determination of asymmetric parameter (γ):

The enrgy ratio$E2_2^+/E2_1^+$ given by equation (6) depends only on γ and this is evaluated by feeding experimental energies $E2_2^+$ & $E2_1^+$ of the two 2^+satates.

$$\frac{E2_2^+}{E2_1^+} = \frac{1+\sqrt{1-\frac{8}{9}X}}{1-\sqrt{1-\frac{8}{9}X}} \tag{6}$$

We denote the parameter obtained from these energies by $\gamma_{e.}$ In an another way the branching ratio $B(E2; 2_2^+ \rightarrow 2_1^+)/B(E2; 2_2^+ \rightarrow 0_1^+)$ which is generally known experimental quantity and is a function of γ alone is used to determine this parameter as –

$$\frac{B(E2;2_2^+\rightarrow 2_1^+)}{B(E2;2_2^+\rightarrow 0_1^+)} = \frac{\frac{10}{7}\left[\frac{X}{9-8X}\right]}{\frac{1}{2}\left(1-\frac{3-2x}{\sqrt{9-8x}}\right)} \tag{7}$$

The parameter so obtained is denoted by $\gamma_{b.}$

(b) Determination of Moment of Inertia (I_0):

The moment of inertia is extracted from equation (4) using the enrgy of first excited 2^+ state and is denoted by I_0

$$I_0 = \frac{J_0}{\hbar^2} = \frac{9-\sqrt{81-72X}}{XE_1^+} \tag{8}$$

Discussion:

The asymmetric parameters γ_e and γ_b are evaluated from experimental level energies and E2 transition rates and are tabulated (Table I). There is no general trend in values of γ with neutron number but, one thing is clear that the values of γ_b and γ_e are almost equal in all of the nuclei of Mo, Ru and Pd. The γ_b values are plotted against its valence nucleon product N_pN_n, an external parameter in fig. 1. The smooth behavior of γ with N_pN_n is surprising. Although, this has been shown before in other regions i.e A~120-140 and A~160-200 [2, 3] but, in A~90-120 region where variation of γ in Ru, Mo and Pd are opposite with N_pN_n when considered individually (Table I).

The asymmetric parameter γ, for those nuclei whose energy of 2_2^+ state are not known, have been predicted from fig.1. This value of γ is used to calculate the energies of 4^+, 6^+ and 8^+ state of yrast band using ARM for some of Ru, Pd and Mo nuclei and are listed in Table II. The systematic study of γ shape variables and moment of inertia I_0 in Mo, Ru and Pd nuclei concludes that the γ values evaluated seperatly from energies and transition rates are found very close to each – other and this gives confidence in the values obtained.

Table I

The values of moment of inertia I_0 and asymmetric parameter γ for even – even Mo, Ru and Pd Nuclei alongwith N_pN_n

Nucleus	R_e	R_b	γ_e	γ_b	I_0
^{92}Mo	2.05	-	28.0	-	3.93
^{94}Mo	2.14	-	27.0	-	6.73
^{96}Mo	2.09	-	28.0	-	7.63
^{98}Mo	2.01	-	29.0	-	6.86
^{100}Mo	1.90	30.25	30.0	26.0	11.21
^{102}Mo	2.86	-	23.0	-	18.28
^{104}Mo	4.22	5.80	19.0	20.0	25.84
^{106}Mo	4.14	4.20	19.0	17.5	28.90
^{98}Ru	2.17	50.00	27.0	26.8	8.98
^{100}Ru	2.52	24.40	24.0	25.5	10.28
^{102}Ru	2.32	26.30	25.5	25.7	12.02
^{104}Ru	2.49	18.52	24.5	24.5	15.65
^{106}Ru	2.93	-	22.5	-	19.84
^{108}Ru	2.93	10.20	22.5	22.3	22.12
^{110}Ru	2.54	14.20	24.0	23.8	23.04
^{112}Ru	2.21	25.30	26.0	25.5	23.98
^{102}Pd	2.76	8.30	23.0	21.5	9.96
^{104}Pd	2.41	20.00	25.0	25.0	10.18
^{106}Pd	2.20	37.40	26.5	26.3	11.35
^{108}Pd	2.15	71.40	27.0	27.3	13.51
^{110}Pd	2.18	71.43	26.7	27.3	15.58
^{112}Pd	2.11	-	28.0	-	17.00
^{114}Pd	2.09	-	28.0	-	17.82
^{116}Pd	2.16	-	27.0	-	17.81

Table – II

Predicted values of energies (in KeV) of 4^+, 6^+ and 8^+ states of yrast band for some Mo, Ru and Pd Nuclei

Nucl	γ	$E2_1^+$	$E4_1^+$	$E6_1^+$	$E8_1^+$
^{84}Mo	24	443.8	1284.8	2393.4	3506.9
^{86}Mo	25	566.6	1606.8	2940.0	4276.1
^{88}Mo	26	740.5	2058.6	3720.2	5435.2
^{90}Mo	30	947.9	2527.7	4860.8	6918.7
^{88}Ru	26	616.0	1713.3	3094.8	4521.4
^{90}Ru	27	738.1	2017.7	3642.7	5317.3
^{92}Ru	28	864.6	2330.8	4229.3	6175
^{94}Ru	30	1430.	3813.3	7330.0	10423.3
^{114}R	23	127.0	375.3	710.0	763.6
^{94}Pd	28	814.2	2195.9	3984.5	5817.6
^{96}Pd	30	1415.	3773.3	7256.1	10328
^{98}Pd	28	863.1	2328.1	4224.3	6167.8
^{100}P	27	665.5	1818.1	3282.4	4792.6
^{118}P	26	378.4	1051.3	1899.1	26

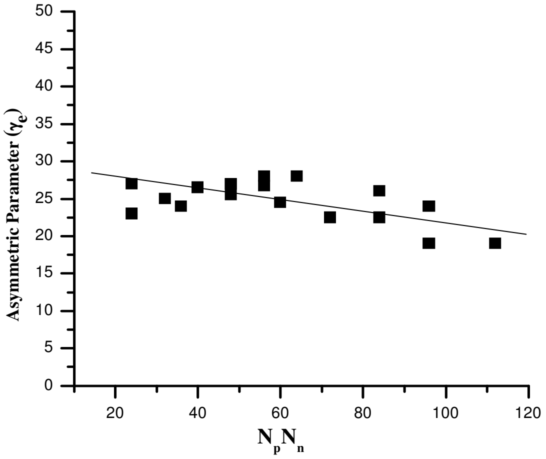

Fig. 1 **The variation γ values plotted against its valence nucleon product N_pN_n for Mo, Ru and Pd nuclei**

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