A general formula for α-decay life-time

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Introduction

In the approach we have proposed recently [1–3] for calculation of Q-value energy and decay half-life $T_{1/2}$ on the α decay of radioactive heavy ions, the α+nucleus system is considered as a Coulomb-nuclear potential scattering problem and the accurately determined resonance energy (E) of the quasibound state is taken as the Q-value of the decaying system. The width or life time of the resonance state accounts for the decay half-life. The normalized regular solution u(r) of the modified Schrödinger equation is matched at radius r=R to the outside Coulomb-distorted outgoing spherical wave $f_C(kr) = G_0(\eta, kr) + iF_0(\eta, kr)$ for ℓ=0,

$$u(r) = N_0[G_0(\eta, kR) + iF_0(\eta, kR)], \qquad (1)$$

where R is the radial position outside the range of the nuclear field.

For a typical α-nucleus system with α particle as the projectile and the daughter nucleus as the target, let A_p and A_t denote mass numbers and Z_p and Z_t stand for charge numbers, respectively, with m representing the reduced mass of the system. Here, the wave number $k = \sqrt{\frac{2m}{\hbar^2}E}$ and η stands for the Coulomb parameter

$\eta = m\frac{Z_pZ_te^2}{\hbar^2 k}$. With this the mean life T (or width Γ) of the decay is expressed in terms of amplitude N_0 as

$$T = \frac{\hbar}{\Gamma} = \frac{m}{\hbar k}\,\frac{1}{\mid N_0\mid^2}. \qquad (2)$$

Since the wave function u(r) decreases rapidly with radius outside the daughter nucleus, it can be normalized by requiring that $\int_0^R \left|u_(r)\right|^2 dr = 1$, Hence T of Eq. (2) is expressed as

$$T = \frac{m}{\hbar k}\frac{\mid G_0(\eta, kR) + iF_0(\eta, kR)\mid^2}{P}, \qquad (3)$$

$$P = \frac{|u(r)|^2}{\int_0^R \left|u_(r)\right|^2 dr}. \qquad (4)$$

In the special cases of the Coulomb-nuclear problem, there are specific values of η and $\rho = kR$ for which $F_0(\eta, \rho)$ and $G_0(\eta, \rho)$ for ℓ=0 can be expressed approximately. In the case where $2\eta > \rho$ for $\ell = 0$ [4],

$$F_0 \approx \tfrac{1}{2}\beta e^{\gamma}, G_0 \approx \beta e^{-\gamma}, \qquad (5)$$

t=$\frac{\rho}{2\eta}$, $\beta = \{t/(1-t)\}^{\frac{1}{4}}$, $\gamma = 2\eta\left\{[t(1-t)]^{\frac{1}{2}} + sin^{-1}t^{\frac{1}{2}} - \frac{1}{2}\pi\right\}$.

Further, in the condition of $2\eta > \rho$, one finds that $|G_0(\eta, \rho)| >> |F_0(\eta, \rho)|$, by several orders of magnitude. Eq. (3) for T reduces to

$$T = \frac{m}{\hbar k}\frac{|G_0(\eta, \rho)|^2}{P}. \qquad (6)$$

Result of the above expression gives values of mean life T or half-life $T_{1/2}$=0.693 T of the decay process at the energy E of resonance considered as the Q-value of decay.

Using an input potential for the α+nucleus system the application of the above potential is carried out. It is seen that in all the cases of successful explanation of the experimental data of $T_{1/2}$ along with Q-value, the values of the quantity P given by (4) is found to be of the order of 2 x 10^{-4}. With this, (6) can be simplified to give the decay half-life as

$$T_{1/2} = 31.696\frac{\sqrt{2\eta\rho}}{4Z_tZ_p}\; exp(-2\gamma)\;\; 10^{-19} sec. \qquad (7)$$

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$\gamma = 2\eta \left\{ [t(1-t)]^{\frac{1}{2}} + sin^{-1} t^{\frac{1}{2}} - \frac{1}{2}\pi \right\}$, t=$\frac{\rho}{2\eta}$.

The matching radius R= $r_0(A_t^{1/3} + A_p^{1/3})$ with $r_0 = 1.35 fm$ fixed for all α+nucleus systems.

We apply the formulation to several cases of α-daughter nuclei. The value of $T_{1/2}$ calculated by using the formula (7) for a given experimental value of Q-value is denoted by $T_{1/2}^{(form)}$ and it is compared with the corresponding experimental result denoted by $T_{1/2}^{(expt)}$. These results for several systems are recorded in Table 1. The results in the table show that the experimental values of $T_{1/2}^{(expt)}$ are extremely large in all the cases considered here and they are fitted very closely by our calculated results given by $T_{1/2}^{(form)}$.

The excellent explanation of the extremely long decay time of α decay process is achieved by the calculated results obtained by using a general formula developed in this work.

TABLE I: Comparison of experimental values [5] of α-decay half-lives and results of present calculation obtained by using formula (7).

Nucleus	$Q^{(expt)}$(MeV)	$T_{1/2}^{(expt)}$(yr)	$T_{1/2}^{(form)}$(yr)
^{140}Nd	2.529	1.03 x 10^8	2.22 x 10^8
^{143}Nd	2.310	1.06 x 10^{11}	1.56 x 10^{11}
^{144}Nd	1.986	7.0 x 10^{15}	2.25 x 10^{16}
^{148}Sm	2.205	1.08 x 10^{14}	2.57 x 10^{14}
^{170}Yb	2.559	2.0 x 10^{15}	2.74 x 10^{15}
^{176}Hf	2.516	1.1 x 10^{18}	3.91 x 10^{17}
^{186}Os	3.243	6.5 x 10^{11}	1.50 x 10^{11}

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