Selective suppression of eigen states with an absorptive delta potential

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Introduction

Delta function potentials are used to explain short range elastic impurities and nature of the spectrum generated by the delta function potential embedded in a box [1] and to construct model atomic systems interacting with the electromagnetic fields leading to multi photon absorption and ionization process [2]. Similar study can be carried out in a nuclear system to analyze different properties of radioactive ions.

We study the resonances generated when particle traverses across two delta potentials in one-dimension (ID). In this contribution, we demonstrate a novel feature that an absorptive delta potential suitably placed within a potential pocket can be used to selectively manipulate and suppress the resonances generated by the pocket.

We first make a study of one-dimensional transmission across potential

$$V(x) = g_1\delta(x)+g_2\delta(x-d/2)+g_3\delta(x-d). \quad (1)$$

The procedure for the calculation of reflection coefficient R for the potential given by (1) is straight forward [3].

Let us consider three delta potentials placed at (1) $x = 0$, (2) $x = d/2$, and (3) $x = d$ according to equation (1). The distance d between potentials 1 and 3 is taken to be d=20 nm. The potential 2 is placed in between 1 and 3. The mass of the particle is taken to be electronic mass m=0.511 x 10^9 meV (milli eV) and the values of strength parameters g_j, j=1, 2, 3, are in (meV nm) unit.

The potential (1) is considered in two alternative scenarios: (i) g_1=4, g_2=0, g_3=4 and (ii) g_1=4, g_2=-1i, g_3=4. The results of reflection coefficient R are plotted in Fig. 1. The minima of R in the plots indicate the energies for the resonance states generated by the potentials.

In the upper panel, the plot indicates four minima of the results of R representing four resonance states with energies E_{r1}=0.892 meV, E_{r2}=3.570 meV, E_{r3}=8.037 meV, and E_{r4}=14.297 meV within the range of energy 0< E < 15 meV for the situation (i) of the potentials. With negative imaginary g_2= -1i in the situation (ii) and placing the same at d=10 nm which is the middle place between $x = 0$ and $x = d$ =20 nm for g_1 and g_3, respectively, the plot of R versus E in the lower panel shows two minima depicting two possible resonance states. On comparison with the states of upper panel for the case (i) of potentials, the first and third states from left present in upper panel are not manifested in lower panel. Thus, some states are suppressed alternatively due to the application of attractive imaginary delta potential placed at the middle point between to side delta potentials which generate the resonances states.

We present a simple way to understand the reason behind this phenomenon of selective suppression of states. Instead of potential given by (1), let us consider a delta potential $V_0\lambda\delta(x)$ embedded in a box of size $\mid x \mid$ < a. As $\delta(x)$ has dimension of inverse length, the parameter λ is expressed in unit of length whereas V_0 is in energy unit. That is, we re-

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place $g_1\delta(x)$ and $g_3\delta(x-d)$ used in (1) by infinitely high walls at x=0 and $x = d$. It is easy to verify that the energy levels generated in this box can be obtained by roots $k = k_n$ of this equation

$$ksin2ka + \lambda V_0 sin^2 ka = 0. \quad (2)$$

The condition can also be written as

$$sinka(2kcoska + \lambda V_0 sinka) = 0. \quad (3)$$

From the above equation we observe that when $V_0 = 0$, the energy eigen values correspond to $2ka = n\pi$, n=1, 2, 3,... and the corresponding wave functions are even with respect to origin for n=1, 3, ... and odd for n=2, 4,... as expected. It is well known that an even state wave function always has non-zero value at the origin. When the delta function potential is introduced at origin, expression (3) shows that the energy levels correspond to ka=$\pi, 2\pi, 3\pi$ ...... remain as before, but the levels corresponding to $ka = \pi/2, 3\pi/2$... are replaced by the energy levels generated by the roots of the equation
$2kcoska+ \lambda\ V_0 sinka$ =0.
This means the so called odd states of the original box potential have unaltered energy eigen values but even states are changed by the presence of delta potential. Now when V_0 is made absorptive by setting V_0 = -iW_0, $W_0 > 0$, real energy eigen values cannot be generated by equation (3) except those odd states generated by $sinka = 0$ condition. We find something similar occurs in the structure of R when reflection or transmission occurs across a two symmetric delta function potential in the presence of absorptive delta function potential introduced in between.

As explained above, with regard to expression (1), if one introduces an absorptive potential at $x = d/2$ several states whose wave functions are non-zero at $x = d/2$ are suppressed. That is, the absorptive delta function potential at the middle is effective in suppressing only those states which are non-zero even states at the middle point $x = d/2$, but ineffective in suppressing the point. This gives the reason for the selective suppression of even states generated by a twin delta potentials in one dimension when an additional strongly absorptive delta function potential is introduced at the middle point of the two. This example shows how appropriate positioning of absorptive delta potential can be used in selectively suppressing certain resonance states.

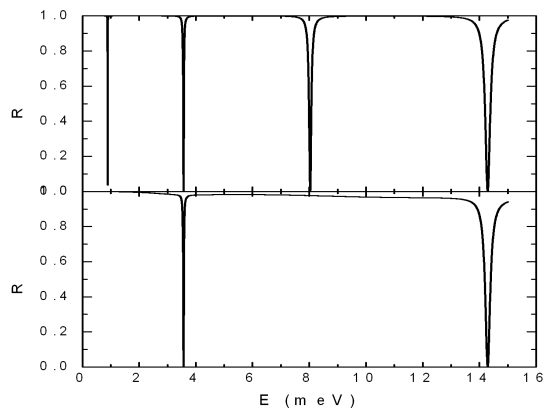

FIG. 1: Variation of R with E for resonance states. Upper panel for situation (i) consisting two delta shells with g_1=g_3=4 meV nm separated by distance d= 20 nm. Lower panel for the situation (ii) consisting three delta shells with g_1=g_3=4 meV nm separated by distance d=20 nm and g_2=-i1 meV nm placed at midpoint d/2=10 nm.

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