Radiative decays of some light mesons with relativistic phase spaces

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Introduction

The $Q\bar{Q}$ potential in the whole range of distances cannot be derived from the first principles of QCD. Despite the substantial progress made in lattice simulations on large computers QCD has not been solved so far. Therefore, the study of hadron spectroscopy is done in terms of the models, which in principle have come from the QCD Lagrangian. Since the exact form of confinement from QCD is not known, phenomenological quark models are built incorporating the salient features of QCD. The constituent models are either non relativistic quark models (NRQM) with suitably chosen potential or relativistic models where the interaction is treated perturbatively. The quark spectroscopy has been described by non relativistic potential models with Buchmuller and Tye potential or a power law potential or a logarithmic potential or a Coulomb plus linear potential reflecting the dynamics expected from QCD. Most of the models have common ingredients. Almost all phenomenological models are based on some variant of the Coulomb plus linear confining potential. Quark potential models typically include one gluon exchange and most of the models also include the running constant of QCD, $\alpha_s(Q^2)$ with relativistic effects included at some level. All models that have been considered here include the spin – dependent effects which arise from the one gluon exchange plus a relativistic spin-orbit Thomas precession term expected of an object with spin (the quark or antiquark) moving in central potential. Potential models have been reasonably successful in describing the most known mesons. This paper studies the radiative decays of certain light mesons using spectroscopic parameters.

Theory

To calculate the transitions betweeen quarkonium levels with emission of a photon the standard multipole expansion is applied since quarkonium here is treated as non relativistic system. In potential model approach , the spatial dependence of EM transition amplitudes reduces to functions of quark position and momentum between the initial and final state wave functions. Expanding the matrix elements in powers of photon momentum generates the electric and magnetic multipole moments and is also an expansion in powers of velocity. The leading order transition amplitudes are the electric dipole (E1) and magnetic dipole (M1) terms, which are given by the corresponding terms in the Hamiltonian

E1 Transitions

The decay rate for transitions from a $^3S_1$ state to ^3P_J state [1, 2] is given by,

$$\Gamma_{(^3S_1\to\gamma^3P_J)}=(2J+1)\frac{4}{27}e_q^2\alpha_{em}k_0^3\left|I_{PS}\right|^2$$

where k_0 is the energy of the emitted photon, e_q is the charge of the quark, α_{em} is the fine structure constant and I_{PS} is the radial overlap integral which has the dimension of length.

$$I_{PS}=\langle P|r|S\rangle=\int_0^\infty r^3R_P(r)R_S(r)dr$$

with $R_{S,P}(r)$ being the normalised radial wave functions for the corresponding states. The transition from ^3P_J levels to a $^3S_1$ level is given by the expression for the rate,

$$\Gamma_{(^3P_J\to\gamma^3S_1)}=\frac{4}{9}e_q^2\alpha_{em}k_0^3|I_{SP}|^2$$

M1 Transitions

The allowed M1 transitions are essentially $^3S_1 \to {}^1S_0$ and $^1S_0 \to {}^3S_1$. The rate for transitions from a $^3S_1$ state to $^1S_0$ state is [1, 2],

$$\Gamma_{(n^3S_1\to\gamma m^1S_0)}=\frac{4}{3m_q^2}e_q^2\alpha_{em}k_0^3|I_{mn}|^2,$$

where I_{mn} is the overlap integral for unit operator between the coordinate wave functions of the initial and the final meson states and m_q is the mass of the quark. The orthonormality of states guarantees, in the limit of zero recoil, that the spatial overlap is one for states within the same multiplet and zero for transitions between multiplets which have different radial quantum numbers.

$$I_{mn}=\int_0^\infty r^2R_{nS}(r)R_{mS}(r)dr$$

For transitions from $^1S_0$ state to $^3S_1$ state the following expression for the rate is used.

$$\Gamma_{(n^1S_0\to\gamma m^3S_1)}=\frac{4}{m_q^2}e_q^2\alpha_{em}k_0^3\left|I_{mn}\right|^2$$

Magnetic transitions flip the quark spin, so their amplitudes are proportional to the quark magnetic moment and therefore inversely proportional to the constituent quark mass. These transitions have $\Delta l = 0$.

Results

We have calculated the M1 transition widths for some light meson states in the long wavelength approximation. These transitions are reported in PDG [3]. In our calculations the experimental values of the meson masses have been used. Table I gives the calculated values of transiton widths for some of the light meson states in non relativistic phase space. Table II gives the calculated values of transition widths in relativistic phase space. The quality of the calculated results reveal that the non relativistic phase space is not a correct prescription for light mesons. Comparison between the photon energy and the mass of the emitting meson reveals that the relativistic phase space is more suited which is seen in our model calculations [3, 4].

Table I. Radiative decay widths of light mesons in non relativistic phase space (keV).

Transition	Expl. value Γ(keV)	Calculated Γ (keV)
$^1S_0$ --> $^3S_1$		
$\eta'(958)\to\rho^0\gamma$	61.31 ± 5.51	260.96
$\eta'(958)\to\omega\gamma$	6.11 ± 0.78	23.38
^3P_J --> $^3S_1$		
$f_1(1285)\to\rho^0\gamma$	1296 ± 295.2	1138.71
$^1P_1$ --> $^1S_0$		
$b_1(1235)\to\pi^+\gamma$	227.2 ± 58.6	780.21

Table II. Radiative decay widths of light mesons in relativistic phase space (keV).

Transition	Expl. value Γ(keV)	Calculated Γ (keV)
$^1S_0$ --> $^3S_1$		
$\eta'(958)\to\rho^0\gamma$	61.31 ± 5.51	71.45
$\eta'(958)\to\omega\gamma$	6.11 ± 0.78	8.53
^3P_J --> $^3S_1$		
$f_1(1285)\to\rho^0\gamma$	1296 ± 295.2	1289.67
$^1P_1$ --> $^1S_0$		
$b_1(1235)\to\pi^+\gamma$	227.2 ± 58.6	239.23

References

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