A variational approach to study a single λ-hypernucleus as a three-body system: example ${}^{9}_{\lambda}Be$ hypernucleus

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Introduction

In our earlier investigations [1], the dynamics of an asymmetric three-body system consisting of one heavy and two identical light particles has been treated in a simple coordinate space variational approach. The method has been successfully applied to study the structural properties of some Borromean two neutron halo nuclei and some even-even double λ-hypernuclei.

We now investigate the ${}^{9}_{\lambda}Be$ hypernucleus as a three-body model consisting of two α-particles (heavy particle) and a λ-particle (light particle). Present preliminary results indicate that the method appears to work in the new configuration also. Structural quantities like the binding energy B_λ, the r.m.s values of $\alpha-\lambda$ separation $< r_{\alpha-\lambda} >$ and the $\alpha-\alpha$ separation $< r_{\alpha-\alpha} >$ have been determined theoretically.

Theory

To simplify the model we assume that the hypernucleus ${}^{9}_{\lambda}Be$ is treated as a three-body system consisting of a λ-particle and two α-particles. The particles are labeled as 1(λ), 2,3(αs). The mass of the λ-particle is M_1 and the mass of an α particle is M_2. r_1, r_2 and r_3 are the distances between the particle pairs 1-2, 1-3 and 2-3. We first remove the centre of mass motion from the Hamiltonian. Then the variational principle for the Schrödinger equation for the internal motion of the three-body system is given by

$$\partial < \Psi|H - E|\Psi >= 0 \qquad (1)$$

where H, E and Ψ refer to the Hamiltonian, energy and the wave function, respectively, for the internal motion.

The wave function for the internal motion of the three-body system is considered as

$$\Psi = \Phi(R) \qquad (2)$$

where R is a new space co-ordinate defined by

$$R(\eta) = (r_1 + r_2 + \eta r_3)/2 \qquad (3)$$

where η is a scaling parameter and it controls the way the wave function depends on r_1, r_2 and r_3.

In addition to R we define two new coordinate variables R_2 and R_3 given by

$$R_2 = r_2 \;\text{ and }\; R_3 = (1+\eta)r_3/2 \qquad (4)$$

After R_2 and R_3 integrations have been performed in Eq.(1), we finally obtain an eigenvalue equation which is,

$$\frac{d^2F}{dR^2} + \Big[\frac{4(\eta^2+5\eta+8)}{D'}E - \frac{V_{eff}(R)}{D} - \frac{15}{4R^2}\Big]F = 0 \qquad (5)$$

where $F(R) = R^{5/2}\Phi(R)$, D and D' are functions of η and the masses of the λ and the α particles. Thus the starting three body problem reduces finally to an effective two-body equation. $V_{eff}(R)$ is a long range attractive potential which emerges through the R_2, R_3 integration of the actual potentials $U(r_1)$, $U(r_2)$ and $V(r_3)$. Hence

$$V_{eff}(R) = \frac{1}{R^5}\int_0^R dR_3 \int_{R-R_3}^{R-\beta R_3} dR_2 R_2 R_3 \qquad (6)$$
$$\times\ (2R - R_2 - \nu R_3)\ [U(r_1) + U(r_2) + V(r_3)]$$

where $\beta = (\eta-1)/(\eta+1)$ and $\nu = 2\eta/(\eta+1)$. Details of the theory are discussed in [1].

Choice of the $\alpha-\lambda$ and $\alpha-\alpha$ potentials
The $\alpha-\lambda$ potentials used are [2, 3]

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$$V^{(1)}_{\alpha\lambda} = -43.976\exp[-(r/1.566)^2],$$
$$V^{(2)}_{\alpha\lambda} = -60.17\exp[-(r/1.273)^2] \quad (7)$$

and $\alpha-\alpha$ (Ali-Bodmer) potentials [4] used are

$$V^{(1)}_{\alpha\alpha} = -110\exp[-(0.65r)^2] - 42\exp[-(0.35r)^2] + V_c,$$
$$V^{(2)}_{\alpha\alpha} = -282\exp[-(0.55r)^2] - 178\exp[-(0.42r)^2] + V_c$$

where $V_c = \frac{4e^2}{r} erf(\frac{\sqrt{3}r}{2R_\alpha})$. The Coulomb interaction between the two α-particles V_c takes into account the finite size of the α-particles assuming these to have a Gaussian density distribution for the proton centre with an r.m.s radius $R_\alpha = 1.44$ fm.

Denoting by $\bar{R}^2$, $\bar{r}_3^2$, $\bar{r}_1^2$ as the mean square distances of R^2, r_3^2 and r_1^2 respectively, these are calculated as follows using the numerical wave function $F(R)$ obtained from the solution of Eq.(5) as

$$\bar{R}^2 = \frac{\int_0^\infty dR R^2 F^2(R)}{\int_0^\infty dR F^2(R)}$$
$$\bar{r}_3^2 = [\frac{8(\eta^2+7\eta+16)}{7(1+\eta)^2(\eta^2+5\eta+8)}]\bar{R}^2$$
$$\bar{r}_1^2 = [\frac{2\eta^4+14\eta^3+42\eta^2+70\eta+64}{7(1+\eta)^2(\eta^2+5\eta+8)}]\bar{R}^2.$$

Numerical Results

TABLE I: Binding energy (B_λ)= $-E_{min}$

$\alpha-\alpha$	$\alpha-\lambda$	η	B_λ	$< r_{\alpha-\lambda} >$	$< r_{\alpha-\alpha} >$
potential	potential		MeV	fm	fm
$V^{(1)}_{\alpha\alpha}$	$V^{(1)}_{\alpha\lambda}$	0.3	6.52	2.78	3.55
	$V^{(2)}_{\alpha\lambda}$	0.25	5.39	2.71	3.53
$V^{(2)}_{\alpha\alpha}$	$V^{(1)}_{\alpha\lambda}$	0.25	6.48	2.94	3.82
	$V^{(2)}_{\alpha\lambda}$	0.2	5.16	2.92	3.86

Conclusion

The experimental value of $B_\lambda(^9_\lambda Be)$ is $6.71\pm$ 0.04 MeV [5]. Our theoretical results using $V^{(1)}_{\alpha}$ are 6.52 MeV for $V^{(1)}_{\alpha\alpha}$ and 6.48 MeV for $V^{(2)}_{\alpha\alpha}$. With $V^{(2)}_{\alpha}$ the theoretical results are lower. These are 5.39 MeV for $V^{(1)}_{\alpha\alpha}$ and 5.16 MeV for $V^{(2)}_{\alpha\alpha}$. However, in a general way it is observed that the mathematically simple variational ansatz used here to study $^9_\lambda Be$ as a three-body model (two heavy and one light particle viz. α, α, λ) appears to be quite successful. In contrast to our earlier studies involving one heavy and two light particles, our experience is that here the dependence of energy on η is less sharp.

References

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