Shear viscosity to entropy density ratio of an interacting pion gas

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1. Introduction

The experimentally measured elliptic flow v_2 of hadrons in Au+Au collision at RHIC, can be interpreted in terms of viscous hydrodynamics with a small value of η/s. This reveals that the produced matter is an almost perfect fluid characterized by a small value of the shear viscosity over entropy density ratio η/s, close to the quantum or KSS bound. Shear viscosity η characterizes how the system dissipates in the presence of flow gradient. Since, in general shear viscosity is inversely proportional to the interaction cross section, the cross section plays the most important role in determining the value of shear viscosity.

Pions form the most important component in a hadronic system. We intend here to calculate the shear viscosity of pion gas in the relativistic kinetic theory approach, in a hot and dense medium. So we need to obtain the energy dependent $\pi\pi$ cross-section using a phenomenological approach which is close to the experimental value and able to incorporate the medium effects.

2. The $\pi\pi$ cross-section with medium effects

To evaluate the $\pi\pi$ cross section we consider the scattering to proceed via ρ exchange for which the invariant amplitude is evaluated using the effective Lagrangian,

$$\mathcal{L}_{\rho\pi\pi} = \frac{ig_\rho}{4}\, Tr[V^\mu, [\partial_\mu\Phi, \Phi]]. \qquad (1)$$

The coupling constant g_ρ=6.05 is fixed from the $\rho \to \pi\pi$ decay width. In order to describe $\pi\pi$ scattering at low energies it is essential to also include σ-exchange diagrams. We use the interaction $\mathcal{L}_{\sigma\pi\pi} = \frac{1}{2} g_\sigma m_\sigma \vec{\pi}\cdot\vec{\pi}\sigma$ with $g_\sigma =$ 2.5. In isospin basis the matrix elements for all possible values of the total isospin of two pions are given by,

$$M_{I=0} = 2g_\rho^2[\frac{s-u}{t-m_\rho^2} + \frac{s-t}{u-m_\rho^2}] + g_\sigma^2 m_\sigma^2[\frac{3}{s-m_\sigma^2+im_\sigma\Gamma_\sigma} + \frac{1}{t-m_\sigma^2} + \frac{1}{u-m_\sigma^2}]$$

$$M_{I=1} = g_\rho^2[\frac{2(t-u)}{s-m_\rho^2+im_\rho\Gamma_\rho(s)} + \frac{t-s}{u-m_\rho^2} - \frac{u-s}{t-sm_\rho^2}] + g_\sigma^2 m_\sigma^2[\frac{1}{t-m_\sigma^2} - \frac{1}{u-m_\sigma^2}]$$

$$M_{I=2} = g_\rho^2[\frac{u-s}{t-m_\rho^2} + \frac{t-s}{u-m_\rho^2}] + g_\sigma^2 m_\sigma^2[\frac{1}{t-m_\sigma^2} + \frac{1}{u-m_\sigma^2}]. \qquad (2)$$

The differential cross-section is then obtained from $\frac{d\sigma}{d\Omega} = \overline{|M|^2}/64\pi^2 s$ where the isospin averaged amplitude is given by $\overline{|M|^2} = \frac{1}{9}\sum(2I+1)\overline{|I|^2}$. The integrated cross-section is plotted as a function of the centre of mass energy which agrees reasonably well with the experimental data, as seen in Fig.1 (dashed line).

In order to obtain the $\pi\pi$ cross section in the medium the ρ widths appearing in the matrix elements are replaced with the corresponding in-medium ones. The effect of the medium on ρ propagation is quantified through its self-energy. The standard procedure is to evaluate this quantity by perturbative methods using effective interactions.

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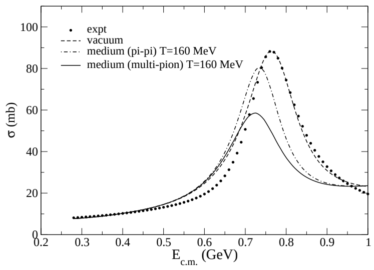

FIG. 1: The $\pi\pi$ cross-section as a function of centre of mass energy.

The self-energy function Π which is related to the decay width by $k_0\Gamma(k) = -Im\Pi$, contain $\pi\pi$, $\pi\omega$, πh_1 and πa_1 loops. The mesons ω, h_1 and a_1 all have substantial 3π and $\rho\pi$ decay widths. The contributions from the loops with those heavy mesons may then be considered as a multi-pion contribution to the ρ self-energy. The cross section obtained by using the in-medium ρ propagator suffers a small suppression of the peak for $\pi\pi$ loop and a larger effect for π-meson loop.

3. The shear viscosity of a pion gas

The relativistic transport equation for the phase space distribution function $f(x,p)$ of a pion gas is written as,

$$p^\mu \partial_\mu f(x,p) = C[f] \quad (3)$$

where $C[f]$ is known as the collision term. For binary elastic collisions $p+k \to p'+k'$ the Uehling-Uhlenbeck collision term is given by

$$C[f] = \int d\Gamma_k \, d\Gamma_{p'} \, d\Gamma_{k'} [f(x,p')f(x,k') \times\{1+f(x,p)\}\{1+f(x,k)\} - f(x,p)f(x,k) \times\{1+f(x,p')\}\{1+f(x,k')\}] \, W (4)$$

where $d\Gamma_q = \frac{d^3q}{(2\pi)^3 q_0}$ and the collision rate W is defined as

$$W = \frac{s}{2}\frac{d\sigma}{d\Omega}(2\pi)^6\delta^4(p+k-p'-k')$$

which includes an explicit factor of 1/2 for indistinguishable particles.

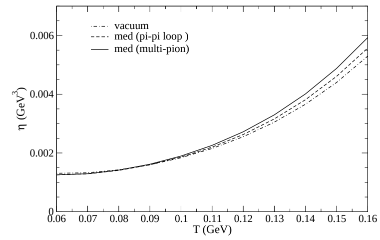

FIG. 2: The shear viscosity as a function of temperature in the Chapman-Enskog approximation.

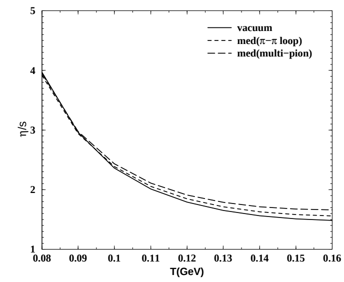

FIG. 3: The shear viscosity to entropy density ratio as a function of temperature.

After solving the relativistic transport equation in Chapmann-Enskog approximation we obtain the expression for shear viscosity,

$$\eta = \frac{T}{10}\,\frac{\gamma_0^2}{c_{00}} \quad (5)$$

where the detailed expression for γ_0 and c_{00} is given in Ref [1]. The latter contains integrals over the $\pi\pi$ differential cross-section which is sensitive to the medium effects.

Fig.2 shows the variation of η with T obtained from equation 5, where the medium effects on the cross section is prominently visible. The variation of η/s with T is shown in the fig.3. The temperature dependence of shear viscosity with and without the medium effect shows a noticeable difference. This reveals that the medium modification of the interaction cross section plays an important role in the evaluation of transport coefficients.

References

[1] S. Mitra, S. Ghosh, S. Sarkar, Phys. Rev. C **85** (2012) 064917.