Topological Dyons in Non-Temporal Gauge

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1. Introduction

Following Dirac's [1] introduction of magnetic monopole to explain the discrete nature of electric charge, Schwinger conceptualized [2] dyons as the particles possessing both electric and magnetic charges. Schwinger's dyons were the Abelian dyons and earlier theories associated with such particles were based primarily on the Dirac's strings [1,3]. Such theories appeared natural because they employed only one potential but the presence of string singularity in the vector potential emerged as their serious drawback. In order to counter this shortcoming, Cabibbo-Ferrari in 1962 [4], proposed a two potential approach to deal with the theories of dyons. The introduction of second potential, a magnetic one, however, allowed the presence of another photon which was called the magnetic photon. The Cabibbo-Ferrari theory therefore appeared to have two photons, and out of the two, the magnetic photon had to be unphysical. Unless a proper explanation comes for the presence of two photons, the Cabibbo-Ferrari two potential approaches too would be inadequate to deal with the particles carrying electric and magnetic charges. One significant explanation comes from Cabibbo and Ferrari themselves that the introduction of second potential gets compensated by the enlargement of the group of gauge transformations. Recently, Singleton [5,6] ,Wilson [7] and Guth [8] have provided substantial support to the Cabibbo-Ferrari two potential approach.

The arguments in favor of two potential approach [5-9] add to the viability of the two-photon theory of dyons. Inspired by these views which lend a great support to two-potential approach, we constructed a Cabibbo-Ferrari type field tensor [Benjwal-Joshi field tensor] [10] to deal with the non-Abelian dyons. Using this field tensor, the temporal gauge dyon solutions have been obtained [11]. In the present paper we have investigated the two potential non-temporal gauge dyon solutions.

2. The Lagrangian density and Field Equations

The system whose gauge group is SU(3)⊗SU(3), is described by the Lagrangian density [11]

$$\mathcal{L} = -\frac{1}{4}A^a_{\mu\nu}A^{\mu\nu a} + \frac{1}{4}\widetilde{B}^a_{\mu\nu}\widetilde{B}^{\mu\nu a} + \frac{1}{2}D^1_\mu\phi^a_e D^{1\mu}\phi^a_e + \frac{1}{2}D^2_\mu\phi^a_g D^{2\mu}\phi^a_g - \varsigma\left(\phi^a_e\phi^a_e + \phi^a_g\phi^a_g - \kappa^2\right)^2 \quad ..1$$

where $A^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - e\ f^{abc}A^b_\mu A^c_\nu$ ..2

$$\widetilde{B}^a_{\mu\nu} = \frac{1}{2}\delta_{\mu\nu\rho\sigma}[\partial^\rho B^{\sigma a} - \partial^\sigma B^{\rho a} - g\ f^{abc}B^{\rho b}B^{\sigma c}] \quad ..3$$

$$D^2_\mu\phi^a_e = (\partial_\mu\delta^{ac} - e f^{abc}A^b_\mu\phi^c_e) \quad ..4$$

$$D^2_\mu\phi^a_g = (\partial_\mu\delta^{ac} - g f^{abc}B^b_\mu\phi^c_g) \quad ..5$$

in which ς and κ are real constants with $\varsigma > 0$. The A^a_μ, B^a_μ the gauge fields, ϕ^a_e and ϕ^a_g denote the Higgs octal fields.

The Euler-Lagrange variations of the Lagrangian density (1) with respect to A^a_μ, B^a_μ, ϕ^a_e and ϕ^a_g, using matrix notation, $A_\mu = e A^a_\mu T^a$, $B_\mu = g B^a_\mu T^a$, $\phi_e = e\phi^a_e T^a$ and $\phi_g = g\phi^a_g T^a$ [12] lead to the field equations

$$\partial_\mu A^{\mu\nu} + i[A_\mu, A^{\mu\nu}] + i[\phi_e, D^{1\nu}\phi_e] = 0 \quad ..6$$

$$\partial_\mu \widetilde{B}^{\mu\nu} + i[B_\mu, \widetilde{B}^{\mu\nu}] + i[\phi_g, D^{2\nu}\phi_g] = 0 \quad ..7$$

$$\partial_\mu D^{1\mu}\phi_e + i\left[A_\mu, D^{1\mu}\phi_e\right] - e\frac{\partial V}{\partial\phi^a_e}T^a = 0 \quad ..8$$

and $\partial_\mu D^{2\mu}\phi_g + i\left[B_\mu, D^{2\mu}\phi_g\right] - g\frac{\partial V}{\partial\phi^a_g}T^a = 0$ ..9

3. The Ansatz

The fields A_μ, B_μ ϕ_e and ϕ_g ansatz [12]

$$\vec{A} = \frac{(1-T_A)}{r^2}\vec{T} - \frac{U_A}{r^3}\vec{U}, \vec{B} = \frac{(1-T_B)}{r^2}\vec{T} - \frac{U_B}{r^3}\vec{U}$$

where $\vec{T} = -\vec{x}\times\vec{\nabla}\hat{\alpha}$, $\vec{U} = -\frac{1}{2}\vec{x}\times\vec{\nabla}\hat{\beta}$ ..10

with $\hat{\alpha} = x^1\lambda^7 - x^2\lambda^5 + x^3\lambda^2 == \hat{\alpha}^a T^a$ ..11

and $\hat{\beta} = \frac{4}{3} I r^2 - 2\hat{\alpha}\hat{\alpha}$ ..12

where, r is the distance three vector with x^1, x^2 and x^3 being the component of the vector and I denoting unit matrix(3x3).

$$A_0 = \frac{R_A}{r^2}\hat{\alpha} + \frac{S_A}{r^3}\hat{\beta} \text{ and } B_0 = \frac{R_B}{r^2}\hat{\alpha} + \frac{S_B}{r^3}\hat{\beta} \quad ..13$$

where R_A, S_A, R_B and S_B are functions of r only.

$$\phi_e = \frac{N_{\phi_e}}{r^2}\hat{\alpha} + \frac{M_{\phi_e}}{r^3}\hat{\beta}, \; \phi_g = \frac{N_{\phi_g}}{r^2}\hat{\alpha} + \frac{M_{\phi_g}}{r^3}\hat{\beta} \quad ..15$$

where the coefficients N and M too are purely r-dependent. We also introduce the three-vectors

$$\vec{P} = \vec{\nabla}\hat{\alpha}, \; \vec{Q} = \frac{1}{2}\vec{\nabla}\hat{\beta}, \; \vec{R} = \vec{x}\,\hat{\alpha} \text{ and } \vec{S} = \vec{x}\,\hat{\beta} \quad ..16$$

4. Finite energy Solutions

To obtain finite energy solutions we express the space-time components of $A^{\mu\nu}$ and $\widetilde{B}^{\mu\nu}$ in terms of the linear combinations of the vectors $\vec{P}, \vec{Q}, \vec{R}, \vec{S}$ and coefficients, where the coefficients are the combinations of T_A, U_A R_A, S_A, N_{ϕ_e} and their counterparts $A \to B$ and $e \to g$. Applying these conditions in eqs.(6)-(9) and employing the condition $T_A = T_B = F_A$ $F_B = M_{\phi_e} = M_{\phi_g} = 0$ we get the set of second order linear differential equations obeying the Prasad- Summerfield solutions that is

$$U_A = \frac{\beta_A r}{\sinh \beta_A r} \quad ..17a$$

$$N_{\phi_e} = \pm \cosh\theta_A (\beta_A r \coth\beta_A r - 1) \quad ..17b$$

$$R_A = \pm \sinh\theta_A (\beta_A r \coth\beta_A r - 1) \quad ..17c$$

$$U_B = \frac{\beta_B r}{\sinh \beta_B r} \quad ..17d$$

$$N_{\phi_g} = \pm \cosh\theta_{\mathbf{B}} (\beta_B r \coth\beta_B r - 1) \quad ..17e$$

$$R_B = \pm \sinh\theta_B (\beta_B r \coth\beta_B r - 1) \quad ..17f$$

The finite energy corresponding to these solutions under non-temporal gauge conditions $A_0^a \neq 0$ and $B_0^a \neq 0$ is

$$m = \frac{16\pi}{e^2}\beta_A \cosh^2\theta_A + \frac{16\pi}{g^2}\beta_B \cosh^2\theta_B \quad ..18$$

where $\beta_A\ \beta_B\ \theta_A$ and $\theta_{\mathbf{B}}$ are arbitrary constants.

5. Electric and Magnetic Charges

The topological electric and magnetic charges obtained corresponding to above $A_0^a \neq 0$ and $B_0^a \neq 0$ conditions are [12]

$$q_e = \pm\frac{2\sinh\theta_A}{e} + \frac{2}{g} \quad ..19$$

and $$q_g = \frac{2}{e} \pm \frac{3\sinh\theta_B}{2g} \quad ..20$$

6. Conclusion

Using a Cabibbo-Ferrari type non-Abelian field tensor, dyon-solutions have been obtained in the non-temporal gauge. Introducing the quantities $\hat{\alpha}$ and $\hat{\beta}$ in terms of Gell-Mann matrices, three-vectors $\vec{P}, \vec{Q}, \vec{R}, \vec{S}, \vec{T}$ and $\vec{U}$ have been defined, the gauge fields have then been expressed in terms of these three-vectors, which result in the second order non-linear field equations whose solutions are similar to the Bogomol'nyi-Prasad Summerfield solutions and whose energy has been shown to be finite. Employing the method discussed in this paper simultaneously both the electric and magnetic charges can be obtained topologically.

References

[1]. P.A.M. Dirac, Proc. Roy.Soc.A133(1931)60.
[2]. J. Schwinger, Phys. Rev. 128 (1962) 2425.
[3]. P. A. M. Dirac, Phys. Rev. 74 (1948) 817; Int. J. Theor. Phys.17(1978) 237.
[4]. N. Cabibbo and E. Ferrari, Nuovo Cimento 23 (1962) 1147.
[5]. D. Singleton, Am. J. Phys. 64 (1996) 452; Int. J. Theor. Phys. 16 (1995) 37.
[6]. A. Kato and D. Singleton, Int. J. Theor. Phys.141 (2002) 1563.
[7]. K. Wilson, Phys. Rev. D 10 (1974) 2445.
[8]. A. Guth, Phys. Rev. D21 (1980) 2291.
[9]. M. N. Saha, Ind. J. Phys.10 (1936) 141; Phys. Rev. 75 (1949) 1968.
[10]. M. P. Benjwal and D.C. Joshi, Phys. Rev. D36 (1987) 629; D. Akers, Int. J. Theor. Phys. 33(1994)1817.
[11]. Vinod Singh, B.V. Tripathi and D. C. Joshi, Ind. J. Pure. Appl. Phys.43(2005)157; 44(2006)567.
[12]. A. Chakrabarti, Ann. Inst. H. Poincare, 23 (1975) 235; E. Kyriakopoulos, Nouvo Cimento A52 (1979) 23.