Wavelet analysis of Ring and Jet-like events at CERN SPS Energy

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The observation of unusually large local-density fluctuations in leptonic, hadronic and nuclear collisions has generated considerable interest in trying to understand the underlying dynamics of multiparticle production in high energy collision. The first detailed event-by-event analysis **[1]** of large statistics data on hadron-hadron interactions was performed to search the dense groups of particles isolated from other particles in the event. The dense groups could look like single dense jets or ring-like events **[2]**. Ring-like structures are observed in the distribution of particles if those are clustered in the narrow region of pseudorapidity (η), but distributed more or less uniformly over the whole azimuth (ϕ) like the spokes of a wheel. If the jet emitting gluons are small then it is more likely that several jets, each restricted to narrow intervals in both (η and ϕ) directions, will be formed, thereby resulting in jet structures in the distributions of final state hadrons. It is the method of Wavelet analysis that reveals the local properties of any pattern in an individual event on various scales. The method fulfills the outlined requirements, providing practical mathematical apparatus to perform multiscale analysis. By choosing the strongest fluctuations, one may chuck out statistical fluctuations and observe those dynamic ones which exceed the statistical component.

In this paper we have performed wavelet analysis to get an idea about the relevant scales and preferred pseudorapidities into two subgroups of particles ring-like and jet-like events for pions multiplicity emitted from ^{16}O-Ag/Br interactions at 60A GeV **[3]**.

In general, the continuous wavelet transform

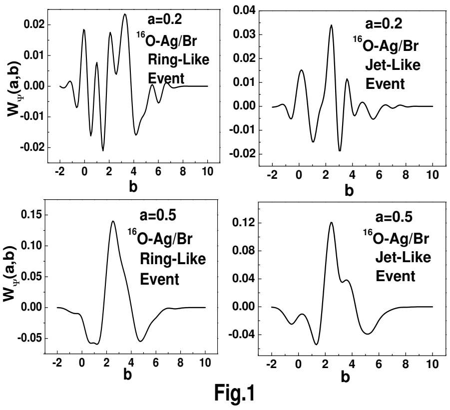

Fig.1

of function f(x) has the form **[4, 5]**:

$$W_\psi(a,b)f = \frac{1}{\sqrt{C_\psi}} \int_{-\infty}^{\infty} f(x)\psi_{a,b}(x)dx \qquad (1)$$

where x is a studied quantity and C_ψ is a normalizing constant. The functions, $\psi_{a,b}(x) = a^{-1/2}\psi(\frac{x-b}{a})$ are shifted and/or dilated derivations of mother wavelet function ψ(x) characterized by translation parameter b and dilation parameter or scale a. The distribution of pseudorapidities can be expressed as $f(\eta) = \frac{dn}{d\eta} = \frac{1}{N}\sum_{i=1}^{N}\delta(\eta-\eta_i)$, where N is the number of particles in a studied data sample and η_i is pseudorapidity of ith particle. Data sample can mean either a few events or one single event or only a part of event. The wavelet transform of the function $f(\eta)$ takes on the form

$$W_\psi(a,b)f = \frac{1}{N}\sum_{i=1}^{N} a^{-1/2}\psi(\frac{\eta_i - b}{a}) \qquad (2)$$

We have studied our analysis using 'Mexican hat' wavelet, which is the second derivative of the Gaussian function and the expression is $g_2 = (1-x^2)e^{-x^2/2}$.

If one study the wavelet η spectrum at the large scale, only coarse features of the pseudorapidity distribution will be pulled out whereas at the small scale the complicated fine

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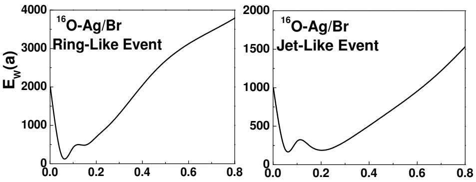

Fig.2

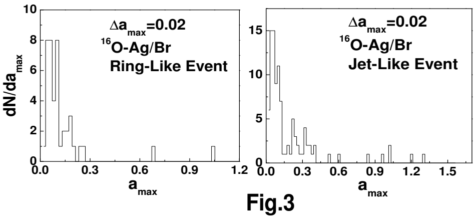

Fig.3

structure will be extracted. Wavelet analysis helps to reveal only pronounced collective flows occurring systematically in many events. Wavelet g_2 pseudorapidity spectras of the ring and jet-like events are presented in Fig.1 for two different scales 'a' (0.2 and 0.5). The maximums in the spectra in Fig. 1 are related to the preferred pseudorapidities of groups of particles. The extent of the areas covered by the local maximums give an estimation of the numbers of particles included to each group. We see that, the nature of the spectrum is different for ring and jet-like events.

Prevailing scales of an event can be obtained by studying the maximums of the scalogram expressed as: $E_W(a) = \int [W_\psi(a,b)]^2 db$. We have shown the scalograms of ring and jet-like events in Fig.2. Each scalogram, shown in the figure, is only for one event taken from each set of events. Scalograms suggest the presence of only one relevant scale in each set of event. The scale range lies between 0.1 to 0.2 and we have focused our interest in that region in our next analysis.

Ring-like structures are expected to occur most likely only at a few distinguished pseudorapidities and are assumed to appear at similar scales in all the events. This preferred pseudorapidities would be manifested by local maximums in the event-by-event wavelet pseudorapidity spectra of large sample of events. If ring-like structures are present in the studied data then distinctive scales and preferred pseudorapidities come into view frequently and systematically in many events but will not appear arbitrarily. This can be investigated by creating the spectra of relevant scales and pseudorapidities pulled out from large sample of events, yet still on the event-by-event basis. Fig.3 presents the distribution of maximum points localized in the scalograms of ring and jet-like events for ^{16}O-Ag/Br interactions. Some relevant scales are identified at 0.03≤a≤0.18 for ring-like events and 0.03≤a≤0.33 for jet-like events.

Now, we want to guess the numbers of dominant scales usually occur in the studied events. To perform the job, we need the distribution of the numbers of local maximums found in the scalograms which is shown in Fig.4. We can see from the figures that, for ^{16}O-Ag/Br interactions, the numbers of the dominant scales for ring and jet-like events are 2 and 1 respectively.

In summary we can say, the dominant scale and preferred pseudorapidities have been found out for ring and jet-like events for ^{16}O-Ag/Br interactions through wavelet analysis.

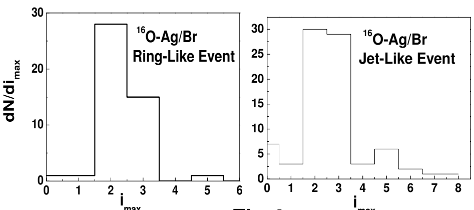

Fig.4

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