

## Numerically modeling Bose-Einstein correlation

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The Bose-Einstein correlation (BEC) between identical bosons is an important feature of multiparticle production. By studying the correlation function one can extract information regarding the dimension of the source from which the final state particles are emitted [1]. The phenomenon is purely quantum statistical in origin – a result of the symmetry of the associated fields. Most of the programming codes that are used to simulate multiparticle production do not include BEC at the correlation level. Hence the problem needs to be modeled independently. One way to do so is to incorporate BEC at the level of so-called ‘after burners’ [1]. Here, we apply one such technique [2] in Au+Au interaction at  $\sqrt{s_{NN}} = 62.4$  and 200 GeV generated by the microscopic transport model, Ultra-relativistic Quantum Molecular Dynamics (UrQMD) [3]. We analyze the simulated data to get some idea about the particle emitting source(s).

The two-particle correlation function of identical particles is defined as the normalized ratio of the two-particle to the product of single-particle distributions as,

$$C_2(q, k) = \frac{N_2(p_i, p_j)}{N_1(p_i)N_1(p_j)}, \quad (1)$$

where  $q = p_i - p_j$  is the relative pair 4-momentum, and  $k = (p_i + p_j)/2$  is the pair average 4-momentum. The functional dependence of  $C_2$  on  $q$  is described by a Gaussian parametrization, and in one-dimension it is

$$C_2(q, k) = 1 + \lambda \exp(-R_{inv}^2 Q_{inv}^2). \quad (2)$$

Here  $Q_{inv}^2 = -q^2$ ,  $\lambda$  is called the correlation (chaoticity) parameter, whereas  $R_{inv}$  charac-

terizes the size of particle producing source, which is assumed to be symmetric.

In order to introduce BEC numerically we follow the prescription of [2]. The algorithm is based on reassigning charges to produced pions keeping their phase space configuration intact. Very briefly, for a single event, the algorithm consists of the following steps:

(a) A charge sign is chosen randomly from the +, - and 0 pool with probability, respectively,  $p_{(+)} = n_{(+)} / n$ ,  $p_{(-)} = n_{(-)} / n$  and  $p_{(0)} = n_{(0)} / n$ . Here  $n_{(+,-,0)}$  is the number of  $\pi^{+,-,0}$ , and  $n = n_{(+)} + n_{(-)} + n_{(0)}$  is the total number of pions in an event. Now the chosen charge sign is assigned to a pion, irrespective of its original charge, selected randomly, say the  $i$ th one, from the event. The  $i$ th pion now defines a phase space cell.

(b) A weight factor  $P_{ij} \in (0, 1)$ ,

$$P_{ij} = \exp \{ -(x_i - x_j)^2 (p_i - p_j)^2 / 2 \}$$

is assigned to another pion, say the  $j$ th. Here  $x_i(x_j)$  are the space-time coordinates of the  $i$ th ( $j$ th) particle.

(c) Pions, like the  $j$ th, are now added to the cell of the  $i$ th one provided  $P_{ij} > r$ , where  $r \in (0, 1)$  is a uniformly distributed random number. However, if  $r > P_{ij}$  and/or for a particular charge variety  $n_{(ch)} = 0$ , one goes back to step (a). The probabilities  $p_{(+)}$  etc. are now updated in terms of the remaining pions.

The procedure continues until all the pions in the event under consideration are re-assigned new charges, and then the process is repeated for other events. Ultimately, the algorithm results in formation of phase-space cells containing identical charges without altering the single particle inclusive distribution.

The UrQMD model is used to generate Au+Au events at  $\sqrt{s_{NN}} = 62.4$  and 200 GeV. The input time parameter `tim` is set to 500

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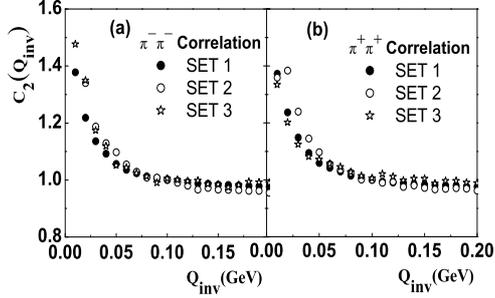


FIG. 1: Correlation functions for identical pion pairs in Au+Au collisions at  $\sqrt{s_{NN}} = 62.4$  GeV. SET 1: minimum bias, SET 2: pions with  $|\eta| < 1$ , SET 3:  $b_{\max} < 4$ ,  $|\eta| < 1$ , pion pairs with  $0.1 < k_t < 0.2$  GeV/c and pair rapidity  $|y_{ij}| < 0.5$ .

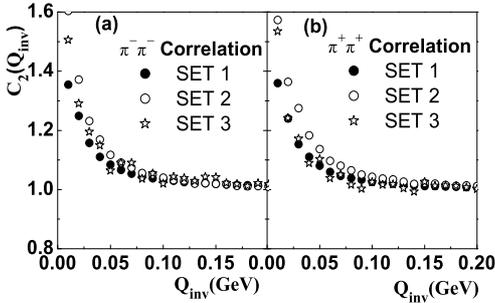


FIG. 2: The same as FIG. 1 but at  $\sqrt{s_{NN}} = 200$  GeV.

fm/c. We use the OSCAR1997A output files of UrQMD which provide the particle ID, its four-momentum, mass and freeze-out four-coordinates. After incorporating BEC into the output data of UrQMD, the  $C_2$  values are obtained for identically charged pion pairs. In FIG. 1  $C_2$  is plotted as a function of  $Q_{\text{inv}}$  at  $\sqrt{s_{NN}} = 62.4$  GeV only for  $\pi^- \pi^-$  and  $\pi^+ \pi^+$  pairs. Corresponding plots at  $\sqrt{s_{NN}} = 200$  GeV are shown in FIG. 2. In these diagrams we use the following notations – SET 1: minimum bias, SET 2: pion pseudorapidity  $|\eta| < 1$ , SET 3: impact parameter  $b_{\max} < 4$ ,  $|\eta| < 1$ ,  $0.1 < k_t < 0.2$  GeV/c and pair rapidity  $|y_{ij}| < 0.5$ . The  $C_2$  functions are fitted to Gaussian functions [Eq. (2) – not shown] and the fit parameters are shown in TABLE I and II, respectively, for  $\sqrt{s_{NN}} = 62.4$  and 200 GeV. The parameter values are comparable to the *rms* nuclear radius of  $^{197}\text{Au}$ .

In summary, implementing BEC by a charge

TABLE I: The Gaussian [Eq. (2)] fit parameters at  $\sqrt{s_{NN}} = 62.4$  GeV.

DATA	$\lambda$	$R_{\text{inv}}$ (fm)
$\pi^- \pi^-$ SET 1	$0.394 \pm 0.057$	$5.448 \pm 0.363$
SET 2	$0.893 \pm 0.067$	$6.475 \pm 0.518$
SET 3	$0.527 \pm 0.019$	$6.067 \pm 0.219$
$\pi^+ \pi^+$ SET 1	$0.388 \pm 0.017$	$5.375 \pm 0.293$
SET 2	$0.475 \pm 0.024$	$4.275 \pm 0.191$
SET 3	$0.316 \pm 0.018$	$5.318 \pm 0.434$
$\pi^0 \pi^0$ SET 1	$0.336 \pm 0.017$	$5.247 \pm 0.280$
SET 2	$0.425 \pm 0.015$	$4.592 \pm 0.174$
SET 3	$0.424 \pm 0.018$	$5.199 \pm 0.235$

TABLE II: The Gaussian [Eq. (2)] fit parameters at  $\sqrt{s_{NN}} = 200$  GeV.

DATA	$\lambda$	$R_{\text{inv}}$ (fm)
$\pi^- \pi^-$ SET 1	$0.332 \pm 0.016$	$5.550 \pm 0.275$
SET 2	$0.579 \pm 0.033$	$6.052 \pm 0.343$
SET 3	$0.487 \pm 0.044$	$5.631 \pm 0.519$
$\pi^+ \pi^+$ SET 1	$0.341 \pm 0.015$	$5.536 \pm 0.291$
SET 2	$0.502 \pm 0.031$	$5.249 \pm 0.335$
SET 3	$0.562 \pm 0.045$	$6.028 \pm 0.578$
$\pi^0 \pi^0$ SET 1	$0.325 \pm 0.019$	$5.757 \pm 0.337$
SET 2	$0.406 \pm 0.013$	$4.616 \pm 0.154$
SET 3	$0.388 \pm 0.015$	$4.956 \pm 0.109$

reassigning algorithm in the UrQMD generated events on Au+Au collisions at  $\sqrt{s_{NN}} = 62.4$  GeV and 200 GeV, we estimate the source radius parameter  $R_{\text{inv}} \sim 4.5 - 6.5$  fm, and the correlation parameter  $\lambda \sim 0.3 - 0.6$ . A more general formulation of the method in terms of the so-called Bertsch-Pratt parametrization [4], is under progress.

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