Next to leading order calculation of neutrino emissivity from degenerate quark matter

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Introduction

In this work, we investigate non-Fermi liquid(NFL) behavior that enters into the emissivity of the neutrinos from the neutron star(NS) composed of degenerate quark matter core. Such calculation was first done in Ref.[1] where the author studied only the Fermi liquid(FL) case. Recently, the calculation was extended to the leading order (LO) NFL effect in Ref.[2]. We shall go beyond the LO to calculate the NFL effects at next to leading order(NLO) that enter into the expression of the emissivity of neutrinos and consequently affect the cooling of the NS.

Formalism

The mean free path(MFP) of the neutrinos is determined by the quark neutrino interaction in dense quark matter *via.* weak processes. For this case, considering the simplest β decay reactions, the neutrino MFP is related to the total interaction rate due to neutrino emission averaged over the initial quark spins and summed over the final state phase space and spins. For the absorption process and it's inverse, MFP is given by[1],

$$\frac{1}{l^{abs}_{mean}(E_\nu,T)} = \frac{g'}{2E_\nu}\int\frac{d^3p_d}{(2\pi)^3}\frac{1}{2E_d}\int\frac{d^3p_u}{(2\pi)^3} \frac{1}{2E_u}\int\frac{d^3p_e}{(2\pi)^3}\frac{1}{2E_e}(2\pi)^4\delta^4(P_d+P_\nu-P_u -P_e)|M|^2\{n(p_d)[1-n(p_u)][1-n(p_e)] +n(p_u)n(p_e)[1-n(p_d)]\}, \quad (1)$$

where, g' is the spin and color degeneracy, considered to be 6. The total emissivity of the non-degenerate neutrinos is obtained by multiplying the neutrino energy with the inverse of the MFP with appropriate factors and integrated over the neutrino momentum. The relation is obtained as[1],

$$\varepsilon = \int\frac{d^3p_\nu}{(2\pi)^3}E_\nu\frac{1}{l(-E_\nu,T)} \quad (2)$$

To evaluate the above quantities we will take into account the quark self energy for the case of degenerate matter. Interactions in the medium severely modify the on-shell self energy of the quarks which is manifested in the slope of dispersion relation for the relativistic degenerate plasma. For quasiparticles close to the Fermi momentum, the one-loop self energy is dominated by soft gluon exchanges. For the calculation of emissivity, one needs to know the modified dispersion relation[3, 4],

$$\omega_\pm = \pm(E_{p(\omega_\pm)} + \mathrm{Re}\Sigma_\pm(\omega_\pm, p(\omega_\pm))) \quad (3)$$

where $\omega_\pm$ denotes the quasiparticle/antiquasiparticle energy. As we are considering only quasiparticles, we will consider only ω_+ and denote it by ω. We now quote the real part of quark self energy as already calculated and notice that no explicit dependence on spatial momentum occurs[4]:

$$\mathrm{Re}\Sigma_+(\omega) = -\mathrm{g}^2\mathrm{C_F}\mathrm{m}\Big\{\frac{\epsilon}{12\pi^2\mathrm{m}}\Big[\log\Big(\frac{4\sqrt{2}\mathrm{m}}{\pi\epsilon}\Big) +1\Big] + \frac{2^{1/3}\sqrt{3}}{45\pi^{7/3}}\Big(\frac{\epsilon}{m}\Big)^{5/3} - 20\frac{2^{2/3}\sqrt{3}}{189\pi^{11/3}}\Big(\frac{\epsilon}{m}\Big)^{7/3} - \frac{6144-256\pi^2+36\pi^4-9\pi^6}{864\pi^6}\Big(\frac{\epsilon}{m}\Big)^3\times \Big[\log\Big(\frac{0.928\,m}{\epsilon}\Big)\Big] + \mathcal{O}\Big(\Big(\frac{\epsilon}{m}\Big)^{11/3}\Big)\Big\} \quad (4)$$

where $\epsilon = (\omega-\mu)$.Thus the modified dispersion relation will be required to calculate

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$dp/d\omega$ needed for the phase space evaluation of the emissivity[5, 6].

Emissivity of the neutrinos

We have calculated the emissivity of non-degenerate neutrinos considering only depopulation of neutrinos as in this case repopulation or the reverse reaction is assumed to be zero. Thus, we arrive at the following results of emissivity of neutrinos, where the total contribution is the sum of the three terms as follows,

$$\varepsilon - \varepsilon_0 = \varepsilon_{LO} + \varepsilon_{NLO} \tag{5}$$

where,

$$\varepsilon_0 \simeq \frac{457}{630} G_F^2 cos^2\theta_c \alpha_s \mu_e T^6 \mu^2 \tag{6}$$

is the usual Fermi liquid contribution which agrees with the result presented in ref.[1]. The LO contribution is given as,

$$\varepsilon_{LO} \simeq \frac{457}{3780} G_F^2 cos^2\theta_c C_F \alpha_s \mu_e T^6 \frac{(g\mu)^2}{\pi^2} \ln\Big(\frac{4g\mu}{\pi^2 T}\Big) \tag{7}$$

which is in agreement with the result quoted in ref.[2]. Now, following the procedure in [2, 5], the NLO contribution to the neutrino emissivity is obtained as[6],

$$\begin{aligned} \varepsilon_{NLO} \simeq & \frac{457}{315} G_F^2 cos^2\theta_c C_F \alpha_s \mu_e T^6 \Big[c_1 T^2 \\ & + c_2 T^{2/3} (g\mu)^{4/3} - c_3 T^{4/3} (g\mu)^{2/3} \\ & - c_4 T^2 \ln\Big(\frac{0.656 g\mu}{\pi T}\Big)\Big] \end{aligned} \tag{8}$$

where c_1, c_2, c_3 and c_4 are constants[6]. It is to be noted here that the NFL correction only appears in the phase space integral of the emissivity of neutrinos[2].

Cooling process via neutrino emission

As the temperature of the neutron star shows a dependency with time, we have analysed the cooling behavior of the star. The specific heat of the degenerate quark matter [7] and the emissivity of the neutrinos has been taken into account. The cooling equation is given as,

$$c_v(T) \frac{\partial T}{\partial t} = -\varepsilon(T) \tag{9}$$

where c_v is also calculated in NLO[7]. The above equation cannot be solved analytically and we have resorted to numerical evaluation. The emissivity of the neutrinos at NLO show considerable increase with respect to the FL result; although there is a marginal increase with the LO result. The cooling of the NS is marginally faster in case of NFL(NLO) as compared with the FL result[6].

Conclusion

We have calculated the expression of emissivity by incorporating NFL effects upto NLO. It is seen that the emissivity contains terms at the higher order which involves fractional powers in (T/μ). We have found that there is an increase in emissivity due to NLO corrections over the FL and LO result. We have also examined the cooling behavior of a neutron star by incorporating NLO correction to the specific heat and emissivity which affect the result considerably compared to the simple FL case.

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