

Systematic for empirical variable $\frac{\epsilon}{\delta}$ with $N_p N_n$ and estimation of B (E2) \uparrow in Ru nuclei

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The proton- neutron interaction has been considered as key ingredient in the development of configuration mixing, collectivity and ultimately deformation in the atomic nuclei. The simple product of the valence proton and valence neutron numbers ($N_p N_n$) is both interpretive in the classification of collective motion of nuclear structure and predicative in investigating the unknown mass region. In 1985 Casten describe the $N_p N_n$ scheme for even-even nuclei, in which $E_{2^+_1}$, $\frac{E_{4^+}_1}{E_{2^+}_1}$ and $B(E2; 0^+_1 \rightarrow 2^+_1)$ values were plotted against the product of valence proton no. and valence neutron no. $N_p N_n$. The systematics for each observable was not only smooth but similar, also from region to region. It was found that the quantity $\frac{\epsilon}{\delta} = \beta_2 / (N_p + N_n)$ which closely correlated with the moment of inertia provides an excellent scaling factor that allows one to access the rapidity of different transition regions and to predict the properties of new nuclei. Moreover the slopes of the different part of systematic are related to the average interaction, per proton neutron pair, in the highly overlapping orbits whose occupation induces structural change [1]. Here ϵ is the quadrupole deformation (similar to β), and δ is the usual pairing gap. The ϵ can be obtained from measured B (E2) values and δ from odd-even mass difference. The $\frac{\epsilon}{\delta}$ reflects the mean field structure and many of the effects that go into the moment of inertia. The quantity δ should be related in some way to the number of like nucleon pair, which is proportional to $N_p N_n$ [2].

Recently, moment of inertia parameter related to triaxial deformation γ and quadrupole deformation β were studied in the $N_p N_n$ Scheme in some medium mass even-even nuclei [3, 4]. In the present work Ru isotopes have been taken to study qualitatively the systematics of various deformation related parameters e.g. $\beta A^{2/3}$, moment of inertia $J = \frac{6}{E_{2^+}_1}$ for asymmetric nuclei and for symmetric nuclei $I_0 = \frac{\epsilon z_1}{E_{2^+}_1}$, $R_{4/2} = \frac{E_{4^+}_1}{E_{2^+}_1}$, quadrupole deformation β , asymmetric parameter γ and a new parameter $\frac{\epsilon}{\delta}$ as a function of $N_p N_n$.

It is clear from table 1 that the values of $\beta A^{2/3}$, J and I_0 continuously increasing in $^{96-112}\text{Ru}$ and not become optimizing at ^{112}Ru as in case of $N_p N_n$. The values of γ are haphazard and irregular with $N_p N_n$. The values of $R_{4/2}$ are decreasing from $^{96-100}\text{Ru}$ and then increases up to ^{108}Ru further decreases at ^{110}Ru and therefore it also remains irregular. The values of $\frac{\epsilon}{\delta}$ are continuously decreasing from $^{96-110}\text{Ru}$, becoming least at mid shell value of $N_p N_n = 96$ and then increases in ^{112}Ru for $N_p N_n = 84$ and hence $\frac{\epsilon}{\delta}$ is the only variable which fruitfully reproduces the qualitative trend of $N_p N_n$.

Getting inspired we have been plotted the systematic of $\frac{\epsilon}{\delta}$ with $N_p N_n$ and shown in Fig 1. The different portions of ascending and descending parts of systematic have different slopes which are due to different interactions among the p-p and p-h pairs. The systematic is used to predict the values of ϵ and

hence $B(E2; 0_1^+ \rightarrow 2_1^+)$ knowing only $N_p N_n$ values for some nuclei in Ru isotopic chain. The predicted values are compared with Bohr

Mottelson, Grodzins estimates and experimental values tabulated in table 2. The predicted values are in good agreement with other estimates as well as experimental one.

Table - 1

Nucl.	$N_p N_n$	β	γ (in deg)	$\beta A^{2/3}$	$R_{4/2}$	J	I_0	$\frac{\epsilon}{\delta} \times 10^{-4}$
⁹⁶ Ru	12	0.153(3)	25	3.31	4.65	7.20	6.56	197(4)
⁹⁸ Ru	24	0.195(3)	27	4.24	2.67	9.20	8.98	195(3)
¹⁰⁰ Ru	36	0.215(1)	24	4.67	2.27	11.12	10.28	179(1)
¹⁰² Ru	48	0.240(2)	25.5	5.33	2.33	12.63	12.02	172(1)
¹⁰⁴ Ru	50	0.271(2)	24.5	6.06	2.48	16.76	15.65	169(2)
¹⁰⁶ Ru	72	0.257(34)	22.5	6.40	2.65	22.21	19.84	143(19)
¹⁰⁸ Ru	84	0.292(22)	22.5	6.71	2.75	24.77	22.14	146(11)
¹¹⁰ Ru	96	0.295(17)	24	6.95	2.65	24.93	23.04	134(7)
¹¹² Ru	84	0.306(13)	26	7.02	7.02	25.35	23.98	153(15)

Table - 2

Nucl.	$N_p N_n$	$\frac{\epsilon}{\delta} \times 10^{-4}$ (exp)	$\frac{\epsilon}{\delta} \times 10^{-4}$ (from fig)	ϵ	$B(E2; 0_1^+ \rightarrow 2_1^+)$			
					Pred.	Bohr Mottelson[5]	Grodzins [5]	Exp.[6]
⁹² Ru	12	-	200	0.160	.243	0.34(14)	0.36(12)	-
⁹⁴ Ru	0	-	215	0.129	.163	0.20(8)	0.21(7)	-
⁹⁶ Ru	12	197(4)	201	0.160	.257	0.35(14)	0.36(12)	0.251(10)
⁹⁸ Ru	24	195(3)	191	0.191	.373	0.44(18)	0.45(15)	0.392(12)
¹⁰⁰ Ru	36	179(1)	180	0.216	.487	0.53(22)	0.54(18)	0.490(5)
¹⁰² Ru	48	172(1)	171	0.239	.623	0.60(24)	0.60(20)	0.630(10)
¹⁰⁴ Ru	60	169(2)	162	0.259	.751	0.79(32)	0.78(26)	0.820(12)
¹⁰⁶ Ru	72	143(19)	155	0.279	.837	1.04(43)	1.01(33)	0.77(20)
¹⁰⁸ Ru	84	146(11)	147	0.284	.950	1.15(47)	1.10(37)	1.01(15)
¹¹⁰ Ru	96	134(7)	140	0.294	1.04	1.15(47)	1.09(36)	1.05(12)
¹¹² Ru	84	153(15)	150	0.300	1.12	1.17(48)	1.09(36)	1.17(23)
¹¹⁴ Ru	72	-	170	0.306	1.18	-	-	-

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