

## Multi-phonon $\gamma$ -vibrational bands in $^{108}\text{Mo}$ nucleus

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### Introduction

In the study of nuclear structure, the multi-phonon  $\gamma$ -vibrational bands are very important subjects. The multiphonon excitations in deformed nuclei can provide valuable information of the nuclear vibrational collectivity. However, to experimentally observe the multi-phonon bands is difficult. So far, considerable efforts have been made to search for the multi-phonon bands i.e.,  $\gamma$ - and  $\gamma\gamma$ -bands in even-even and odd-mass nuclei. If the  $K$  is a band-head quantum number of a quasiparticle band, for the  $\gamma$ -states, there exist two states with  $K^{+2}$  and  $K^{-2}$ . For the  $\gamma\gamma$ -states, there may exist three states with  $K^{-4}$ ,  $K$  and  $K^{+4}$ . It is also expected that the  $K^{+2}$  for  $\gamma$ -state and the  $K^{+4}$  for  $\gamma\gamma$ -state may be easy to be observed, and others may be difficult. Several low lying  $K^\pi = 4^+$  two-phonon  $\gamma\gamma$ -bands have been identified, such as in  $^{166,168}\text{Er}$ ,  $^{164}\text{Dy}$  and  $^{232}\text{Th}$ . In  $A \sim 110$  neutron-rich nuclear region, the one-phonon  $\gamma$ - and  $\gamma\gamma$ -bands have been observed in even-even  $^{104,106,108}\text{Mo}$  and  $^{108,110,112}\text{Ru}$  [1]. For the odd- $A$  nuclei, the study of the  $\gamma\gamma$ -bands is scarce. Recently, in  $A \sim 110$  neutron-rich region, a first  $\gamma\gamma$ -band in an odd- $A$  nucleus has been observed in  $^{105}\text{Mo}$ ,  $^{107}\text{Tc}$  and  $^{103}\text{Nb}$  nuclei.

On the microscopic front, however, there are very few models capable of describing the multi-phonon excitations in a unified manner, in particular, at higher angular momenta. Nuclei around  $A \sim 110$  exhibit some of the most

interesting features in the nuclear periodic table. For instance, some nuclei in this region depict quite large deformation with  $\beta \sim 0.45$  and is understood as due to the reinforcing effect of proton and neutron deformed shell gaps at  $Z=38, 40$  and  $N=60, 62$  [5]. Since, in some nuclei in this region, well developed  $\gamma$ - and  $\gamma\gamma$ -bands have been observed up to quite high angular momenta. But although, yrast bands in this region have been studied using theoretical approach of total routhian surface analysis [5], but there appears no systematic investigation of the  $\gamma$ -bands in this mass region.

The multi-quasiparticle TPSM approach has been employed to investigate the high-spin band structures in Er-isotopes and in the mass  $\sim 130$  region [6]. It has been demonstrated in these studies that  $\gamma$ -bands are built on each intrinsic configuration of the mean-field potential and generalizes the well known surface  $\gamma$ -vibration in deformed nuclei built on the ground-state configuration. Recently, multi-quasiparticle triaxial projected shell model (TPSM) approach has been developed and it has been shown to provide a coherent and accurate description of yrast-,  $\gamma$ - and  $\gamma\gamma$ -bands in transitional nuclei [7]. Quite recently, we have further generalized the TPSM approach to study the  $\gamma$ -vibration in odd-proton nuclei and a preliminary application of this new development for  $^{103}\text{Nb}$  has already been reported [8].

### TPSM Results

For the even-even system, the TPSM basis are composed of 0-qp vacuum, two-proton, two-neutron and the four-quasiparticle con-

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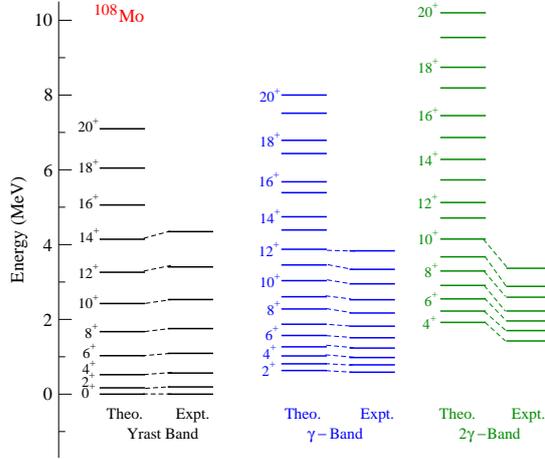


FIG. 1: (Color online) Comparison of experimental and the calculated band energies for  $^{108}\text{Mo}$ . (Data is taken from ref. [1]).

figurations. As in the PSM calculations, we use the pairing plus quadrupole-quadrupole Hamiltonian [9].

$$\hat{H} = \hat{H}_0 - \frac{1}{2}\chi \sum_{\mu} \hat{Q}_{\mu}^{\dagger} \hat{Q}_{\mu} - G_M \hat{P}^{\dagger} \hat{P} - G_Q \sum_{\mu} \hat{P}_{\mu}^{\dagger} \hat{P}_{\mu}. \quad (1)$$

The corresponding triaxial Nilsson mean-field Hamiltonian, which can be obtained by using the Hartree-Fock-Bogoliubov (HFB) approximation, is given by

$$\hat{H}_N = \hat{H}_0 - \frac{2}{3}\hbar\omega \left\{ \epsilon \hat{Q}_0 + \epsilon' \frac{\hat{Q}_{+2} + \hat{Q}_{-2}}{\sqrt{2}} \right\}, \quad (2)$$

where  $\hat{H}_0$  is the spherical single-particle shell model Hamiltonian, which contains the spin-orbit force. The second, third and fourth terms in Eq. (1) represent quadrupole-quadrupole, monopole-pairing, and quadrupole-pairing interactions, respectively.

The TPSM band structures obtained after band mixing are compared with the known experimental data for the even-even  $^{108}\text{Mo}$  nucleus in Fig. 1. The experimental data for  $^{108}\text{Mo}$  is quite rich with both  $\gamma$ - and  $\gamma\gamma$ -bands known up to quite high angular-momenta. It

is quite evident from the comparison that TPSM reproduces, yrast-,  $\gamma$ - and  $\gamma\gamma$ -bands surprisingly well. However, there appears a discrepancy between the theory and the experimental band head energies for the  $\gamma\gamma$ -band. This problem has also been noted in the TPSM study of  $^{103}\text{Nb}$  [8].

In the present work the study of the band structures observed in neutron-rich  $^{108}\text{Mo}$  nucleus has been performed using the recently developed multi-quasiparticle TPSM approach. The advantage of the TPSM approach is that it provides a unified description of the collective and the quasiparticle excitation modes. In this mass region, rich band structures have been observed and this is one of the few regions where  $\gamma$ - and  $\gamma\gamma$ -bands have been observed up to quite high-spin. It has been demonstrated that, in general, TPSM results are in good agreement with the available experimental data for the energies. However, there appears to be a problem to reproduce the band head of the  $\gamma\gamma$ -band for studied nucleus and this problem was also noted in our earlier investigation for  $^{103}\text{Nb}$  [8]. At this stage, there is no simple explanation for this discrepancy and more investigations are needed to understand it.

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