# Comparative study of halo and double-$\Lambda$ hypernuclei in the frame work of three-body model using a variational approach

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### Introduction

In this work ,we show how to treat the dynamics of an asymmetric three-body system consisting one heavy core particle and two identical light extra core particles in a simple coordinate space variational approach [1]. This method gives an efficient way of resolving a three-body system to an effective two body system. It has been successfully applied to study the structural properties of some Borromean two neutron halo nuclei and some even-even double-$\Lambda$ hypernuclei.

Structural quantities like binding energy ,r.m.s. value of core-particle, and particle-particle separation have been determined theoretically. Our results regarding halo nuclei $He^6$, $Be^{14}$, $B^{17}$ and double-$\Lambda$ hypernuclei $_{\Lambda\Lambda}He^6$, $_{\Lambda\Lambda}C^{14}$, $_{\Lambda\Lambda}O^{18}$ are compared.

### The three body model

We assume that the three-body system representing the core and two extra core particles is composed of three spinless particles. The particles are labelled as 1(core), 2,3 (neutrons or lambdas). The wave function for the internal motion of the three-body system is considered as

$$\Psi(r_1, r_2, r_3) = \Phi(R). \tag{1}$$

The new space coordinate $R$ is defined as $R = (r_1 + r_2 + \eta r_3)/2$ where $r_1$, $r_2$, $r_3$ are the distances between the particle pairs 1-2, 1-3 and 2-3 respectively. 1 refers to the core and 2,3 refer to the two neutrons or $\Lambda$ particles. Scaling parameter $\eta$ controls the way the wave function depends on $r_1$, $r_2$, $r_3$. Defining $F(R) = R^{5/2}\Phi(R)$, the problem simplifies to an effective two-body equation in $F(R)$ along with a long range potential $V_{eff}(R)$ given by

$$\frac{d^2F}{dR^2}+\Big[\frac{4(\eta^2+5\eta+8)}{D'}E-\frac{V_{eff}(R)}{D}-\frac{15}{4R^2}\Big]F=0 \tag{2}$$

Numerical solution of Eq.(2) gives the eigen value $E$ and $F(R)$. $D'$, $D$ are the known function of $\eta$ and the masses of the core and the extra core particles.

### Halo nuclei

Neutron halo nucleus contains two loosely bound valence neutron which shows anomalously large size not governed by the $r_0A^{1/3}$ rule. In literature such nuclei are called borromean Halo nuclei because they are bound with only one bond state but have no bound state in the binary subsystems inathree-body model.Some recent reviews on neutron halo nuclei by its discoverer, Tanihata [2] describe how these nuclei opened up a rich new vein of nuclear physics.

The core-n potential used are Woods-Saxon potential and n-n potential are square well and one term Gaussian [3–5]. The results of the calculations are provided in the Table-I. The quantity $r_m$ denotes the matter radius for halo nucleus.

### Hypernuclei

The $\Lambda - \Lambda$ potential 3-G2 [6] and 3G3 [7] has been fitted with the new $B_{\Lambda\Lambda}$ value of $_{\Lambda\Lambda}He^6$ which acts as a constraint on the potential. In continuation of our recent work on double-$\Lambda$ hypernuclei, we report here our study on $_{\Lambda\Lambda}He^6$. The core is even-even for all the double-$\Lambda$ hypernuclei listed here. The $\Lambda$-separation energy $B_\Lambda$, has been calculated using the empirical formula

$$B_\Lambda(A) = 27.0 - 81.9A^{-2/3} \pm 1.5. \tag{3}$$

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TABLE I: Results of the present calculations for halo nuclei.

| Halo Nuclei | Core-n potential Form | n-n potential Form | $B_{2n}$ (MeV) | $< r_{nc} >$ (fm) | $< r_{nn} >$ (fm) | $r_m$ (fm) | $B_{2n}$ other method [2] (MeV) |
|---|---|---|---|---|---|---|---|
| $^6$He | Wood-Saxon | Square well | 0.93 | 4.43 | 4.93 | 2.45 | 0.97 |
| | | Gaussian | 0.94 | 4.34 | 4.495 | 2.42 | |
| $^{14}$Be | Wood-Saxon | Square well | 1.32 | 4.417 | 4.9 | 3.1 | 1.28 |
| | | Gaussian | 1.31 | 4.408 | 4.893 | 3.1 | |
| $^{17}$B | Wood-Saxon | Square well | 2.30 | 4.1 | 4.554 | 2.94 | 2.45 |
| | | Gaussian | 2.29 | 4.1 | 4.54 | 2.94 | |

TABLE II: Results of the present calculations for double-Λ hypernuclei.

| Double-Λ Hypernuclei | $B_\Lambda$ (MeV) | Λ-Λ (Potn.) | $B_{\Lambda\Lambda}$ (MeV) | $< r_{core-\Lambda} >$ (fm) | $< r_{\Lambda\Lambda} >$ (fm) | H-H method [8] $B_\Lambda$ (MeV) | H-H method [8] $B_{\Lambda\Lambda}$ (MeV) |
|---|---|---|---|---|---|---|---|
| $^{6}_{\Lambda\Lambda}He$ | 2.94 | 3G2 | 7.476 | 2.815 | 3.353 | 3.12 | 10.80 |
| | | 3G3 | 7.425 | 2.787 | 3.204 | | |
| $^{14}_{\Lambda\Lambda}C$ | 11.21 | 3G2 | 24.951 | 2.244 | 2.72 | | |
| | | 3G3 | 25.093 | 2.230 | 2.752 | 11.23 | 28.43 |
| $^{18}_{\Lambda\Lambda}O$ | 14.60 | 3G2 | 31.513 | 2.210 | 2.679 | 14.6 | 35.3 |
| | | 3G3 | 31.646 | 2.198 | 2.711 | | |

where $A$ is mass number of the core. The core-Λ potential is considered to be Woods-Saxon

$$V_{core-\Lambda} = -\frac{V_0}{1+\exp(\frac{r-c}{a})} \quad (4)$$

where $c$=$r_0 A^{1/3}$, $r_0$=1.128+0.439A$^{-2/3}$ fm and A is the mass number of the core.

Using this potential in the Schrodinger's equation for the core and the Λ particle, the differential equation is numerically solved to determine the Λ-separation energy. $V_0$ and $a$ are adjusted till the Λ-separation energy so obtained tallies with the value determined from the empirical relation of Eq.(3). The values of $V_0$ and $a$ thus obtained lie within the limits of the best fit values predicted by the RMF calculation [9]. Core mass is calculated using the relation

$$\text{Mass excess} = (\text{Mass} - A)\chi_0 \quad (5)$$

where $\chi_0$= (93148±5) keV and mass excess is tabulated in mass tables. With the core-Λ and the Λ-Λ potentials $\eta$ is varied to get $E_{min}$. Here $B_{\Lambda\Lambda} = -E_{min}$. We have achieved convergence in the two-Λ separation energy $B_{\Lambda\Lambda}$ for each hypernucleus. From the function $F(R)$, the r.m.s values of core-Λ separation $< r_{core-\Lambda} >$ and the Λ-Λ separation $< r_{\Lambda\Lambda} >$ are calculated. The results are given in the Table-II.

**Summary and Conclusion**

The mathematically simple variational ansatz presented above to study halo and double lambda hypernuclei proved the usefulness of special set of coordinates which may be useful to study other asymmetric three particle problems. Our model systems are at best only approximations of real system in nature but they are useful in extracting gross properties of physical relevance.

**References**


[1] S. Mahapatra, J. Nag, Pramana J.Phys. **G60**,1179 (2003).
[2] I. Tanihata, J. Phys. **G22**, 157 (1996).
[3] M.V. Zhukov et al., Phys.Rep. **231**, 151 (1993).
[4] M.V. Zhukov et al., Phys. Lett. **B265**, 20 (1991).
[5] L. Johansen, A.S. Jensen and P.G. Hansen, Phys. Lett. **B244**, 357 (1990).
[6] T. Motoba, H. Bando, T. Fukuda and J. Zofka, Nucl. Phys. **A534**, 597 (1991).
[7] E. Hiyama, M. Kamimura, T. Motoba, T. Yamada, Y. Yamamoto, Prog.Theor. Phys. **97**, 881 (1997).
[8] M.D. Abdul khan, T.K. Das, Pramana J.Phys. **56**, 57 (2001).
[9] Y. K. Gambhir, Nuclear physics at intermediate energies: Lectures of III SERC School on nuclear physics edited by S. Pal (Narosa Publishing House,1999) p.89.