

Relativistic analysis of differential cross-section for $p+^{40}\text{Ca}$ & ^{208}Pb elastic scattering at 200 MeV

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Introduction

Here, we are going to analyze the $p+^{40}\text{Ca}$ and $p+^{208}\text{Pb}$ elastic scattering at 200 MeV under the realm of relativistic Dirac approach. Our purpose is to investigate the effects on the fitting of the differential cross-section ($d\sigma/d\Omega$), obtained using different nuclear ground state densities, for $p+^{40}\text{Ca}$ & ^{208}Pb elastic scattering at 200 MeV.

To solve the elastic scattering we first generate the real scalar (U_s) and the time-like component of the Lorentz four-vector (U_0) potentials for three different nuclear ground state densities, in case of ^{40}Ca and ^{208}Pb , using folding model [1]. These densities for both the targets are represented as LRAY, CHMX (as taken in ref. [2]) and NEG [3]. The LRAY and CHMX densities are the experimental densities which incorporate the medium corrections whereas the NEG densities are the microscopic point-like nuclear matter densities. The obtained potentials assume the shape of the densities which is a two parameter Woods-Saxon form.

The relativistic Dirac equation is simplified to an equivalent Schrödinger equation containing complex central (U_{eff}) and spin-orbit (U_{so}) optical potentials [2]. These U_{eff} and U_{so} potentials are further written in terms of the complex Lorentz scalar (U_s) and vector (U_0) potentials. The real parts of U_s and U_0 potentials are obtained through the folding procedure, where the matter densities of the target nucleus and the parameterized effective nn interactions are used as an input [2]. The respective imaginary parts of these potentials are directly taken from ref. [4].

Formalism

The Dirac equation is used in the mean field approximation in which the nucleon

(meson) fields are replaced by their expectation values. Proton-nucleus scattering is then described using isoscalar-scalar and isoscalar-vector mean fields. Here, these are taken, respectively, as a spherically symmetric complex Lorentz scalar potential, U_s , corresponding to the σ meson field and a spherically symmetric complex Lorentz vector potential, U_0 , corresponding to the ω meson field, together with a spherically symmetric Coulomb potential V_c . With this scalar-vector interaction the Dirac equation becomes ($\hbar = c = 1$)

$$[\vec{\alpha} \cdot \vec{p} + \beta(m + U_s)]\Psi = [E - U_0 - V_c]\Psi, \quad (1)$$

where Ψ is a four-component Dirac spinor with upper and lower components Ψ_U and Ψ_L respectively, E is the total energy of the scattered nucleon in the c. m. frame, $\vec{\alpha}$ and β are Hermitian operators. The second order reduction for the upper component Ψ_U yields,

$$\left\{ p^2 + U_{\text{eff}} + U_{\text{so}} \left[(\vec{\sigma} \cdot \vec{L}) - i(\vec{r} \cdot \vec{p}) \right] \right\} \Psi_u = [(E - V_c)^2 - m^2] \Psi_u. \quad (2)$$

This can further be reduced to the form [2]

$$\left[p^2 + 2E(U_{\text{eff}} + U_{\text{so}}) \right] \phi(\vec{r}) = \left[(E - V_c)^2 - m^2 \right] \phi(\vec{r}), \quad (3)$$

where effective central and spin-orbit potentials are given as

$$U_{\text{eff}} = \frac{1}{2E} [2EU_0 + 2mU_s - U_0^2 + U_s^2 - 2V_cU_0], \quad (4)$$

and

$$U_{\text{so}} = -\frac{1}{2E} \left[\frac{1}{r} \left(\frac{1}{E + m + U_s - U_0 - V_c} \right) \times \frac{\partial}{\partial r} (U_s - U_0 - V_c) \right] \quad (5)$$

Result and Discussion

Differential cross-sections are calculated for $p+^{40}\text{Ca}$ & ^{208}Pb elastic scattering at 200 MeV, corresponding to the three different nuclear ground state densities, using relativistic Dirac phenomenology. These differential cross-sections are obtained treating the strength of the complex scalar and vector potentials (i.e. $\text{Re}U_s$, $\text{Im}U_s$, $\text{Re}U_0$ and $\text{Im}U_0$) as free parameters in chi-square fitting but the geometry of these potentials is constrained to their inherent values. These potentials constitute the central (U_{eff}) and spin-orbit (U_{so}) parts (eq. 4 & 5), which are the main ingredients in the Schrödinger equivalent equation (eq. 3). It can then be solved for the desired scattering observable.

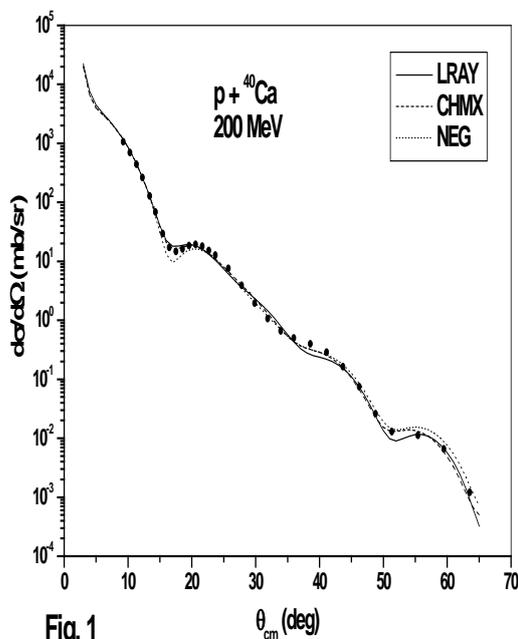


Fig. 1

It is obvious from Fig. 1 that the $p+^{40}\text{Ca}$ differential cross-section data at 200 MeV is adequately reproduced using all the three different nuclear ground state densities without any major distinction. But when the same procedure is applied to reproduce the differential cross-section data for $p+^{208}\text{Pb}$ elastic scattering at the same energy it fails, particularly at higher angles, Fig. 2. This inadequate reproduction of the differential cross-section data for $p+^{208}\text{Pb}$ elastic scattering at 200 MeV suggests its

sensitivity to the geometry of the Lorentz scalar (U_s) and vector (U_0) potentials which has been constrained in our fitting. It does not seem so in case of $p+^{40}\text{Ca}$ elastic scattering where data is satisfactorily reproduced for all ^{40}Ca densities considered here.

Moreover, the numerically calculated differential cross-sections, obtained using different nuclear ground state densities for $p+^{40}\text{Ca}$ & ^{208}Pb elastic scatterings, are however quite close to each other (see Figs. 1 & 2). Although, the LRAY and CHMX densities used in their calculations incorporate the medium corrections but the NEG densities do not. So, it seems that here the medium corrections are not playing any major role in the calculation of differential cross-section for $p+^{40}\text{Ca}$ & ^{208}Pb elastic scattering at 200 MeV.

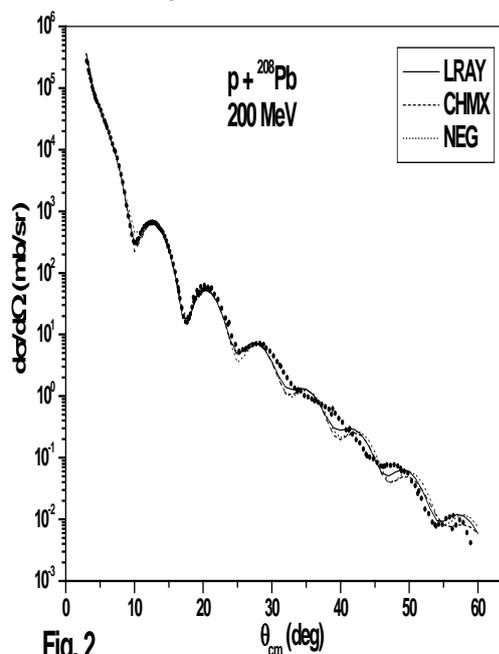


Fig. 2

References

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