New form of Geiger-Nuttall law in α-radioactivity

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Introduction

The Geiger-Nuttall (GN) law is a famous age old formula which explain the measured values of half-life of α-decay with a mathematical characteristic of linear variation of logarithmic half-life with inverse square root of Q-value in $l = 0$ state of transition or decay. Geiger-Nuttall plots for α-decay are expressed as log$T_{1/2}$=a$Q^{-1/2}$+b, since different expression have been proposed [5, 7] to calculate log$T_{1/2}$ from A, Z and the Q-value. The adjustment of the formula coefficients a and b is realized on the total experimental results of α-decay half-lives.To explain half-life of emitted particle carrying some angular momenta,we derive an analytical expression for the decay half-life akin to GN law by considering the unstable parent nucleus as a quantum two-body system of the ejected α- particle and the daughter nucleus exhibiting resonance scattering phenomena under the combined effect of nuclear, Coulomb and centrifugal forces. The formula coefficients in our expression for the half-life are derived naturally and the angular momentum dependence is found inbuilt in the formulation.

An approach have been proposed recently [1–4] for calculation of Q-value energy and decay half-life $T_{1/2}$ on the α decay of radioactive heavy ions, the α+nucleus system is considered as a Coulomb-nuclear potential scattering problem and the accurately determined resonance energy (E) of the quasibound state is taken as the Q-value of the decaying system. The width or life time of the resonance state accounts for the decay half-life. The normalized regular solution u(r) of the modified Schrödinger equation is matched at radius r=R to the outside Coulomb Hankel outgoing spherical wave $f_C(kr) = G_l(\eta, kr) + iF_l(\eta, kr)$ such that

$$u(r) = N_0[G_l(\eta, kR) + iF_l(\eta, kR)], \qquad (1)$$

where R is the radial position outside the range of the nuclear field.

For a typical α-nucleus system with α particle as the projectile and the daughter nucleus as the target, let μ represent the reduced mass of the system and the wave number $k = \sqrt{\frac{2\mu}{\hbar^2}E}$ and η stands for the Coulomb parameter

$\eta = \mu\frac{Z_e Z_d e^2}{\hbar^2 k}$. With this the mean life T (or width Γ) of the decay is expressed in terms of amplitude N_0 as

$$T = \frac{\hbar}{\Gamma} = \frac{\mu}{\hbar k}\,\frac{1}{\mid N_0 \mid^2}. \qquad (2)$$

Since the wave function u(r) decreases rapidly with radius outside the daughter nucleus, it can be normalized by requiring that $\int_0^R \left|u_(r)\right|^2 dr = 1$. Further, using the fact that for a value of radial distance sufficiently large, the value of $G_l(\eta, kR)$ is very large as compared to $F_l(\eta, kR)$ by several order of magnitude, the T of Eq. (2) is expressed as

$$T = \frac{\mu}{\hbar k}\frac{\mid G_l(\eta, kR) \mid^2}{P}, \qquad (3)$$

where

$$P = \frac{\left|u(r)\right|^2}{\int_0^R \left|u_(r)\right|^2 dr}. \qquad (4)$$

Result of the above expression gives values of mean life T or half-life $T_{1/2}$=0.693 T of the decay of the charged particle carrying angular momentum l with Q-value equal to the resonance energy E. In this article, we simplify the formula (3) and put it in the well known linear

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form of GN law for the variation of log$T_{1/2}$ as a function of Q-value of the particle emitted with some amount of angular momentum l.

In the special case of Coulomb-nuclear problem, there are specific values of η and $\rho = kR$ for which the Coulomb Hankel function G_l can be expressed in some simple mathematical form giving quite accurate result. Using $2\eta > \rho$ with $l \geq 0$ [8], and after simplification we get the final formula for $log_{10}T_{1/2}$ is given by

$$log_{10}T_{1/2} = a'\chi' + b'\rho' + c + d, \qquad (5)$$

$\chi' = Z_eZ_d\sqrt{\frac{A}{Q}}$, $\rho' = \sqrt{AZ_eZ_d(A_e^{1/3} + A_d^{1/3})}$, where A$=\frac{A_eA_d}{A_e+A_d}$.The constants a', b', c, and d are expressed as

$a' = 2a_0e^2\sqrt{2m}/\hbar\ Ln10$,
$b' = -b_f\sqrt{2me^2r_0}/\hbar\ Ln10$,
$c = Ln\ c_f/Ln10$,
$d = -[\frac{2}{2\eta^2+1} + \frac{8}{2\eta^2+4} + ...+ \frac{2\ l^2}{2\eta^2+l^2}]/Ln10$
+Ln $M_l/Ln10$,

$$b_f = 2 + a_0 - 2a_1 + (\tfrac{a_0}{4} + a_1 - 2a_2)t^{1/2} + (\tfrac{a_0}{8} + \tfrac{a_1}{4} + a_2 - 2a_3 - 1)t + (\tfrac{5}{64}a_0 + \tfrac{a_1}{8} + \tfrac{a_2}{4} + a_3)t^{3/2} + (\tfrac{5}{64}a_1 + \tfrac{a_2}{8} + \tfrac{a_3}{4} - \tfrac{1}{4})t^2 + (\tfrac{5}{64}a_2 + \tfrac{a_3}{8})t^{5/2} + (\tfrac{5}{64}a_3 - \tfrac{1}{8})t^3$$

$c_f = [\frac{0.693}{3P}\sqrt{\frac{m}{2e^2}A(A_e^{1/3} + A_d^{1/3})r_0/Z_cZ_d}\ \ 10^{-23}]$
$t = \rho/2\eta < 1$
$a_0 = 1.5707288$, $a_1 = -0.2121144$, $a_2 = 0.074240$, $a_3 = -0.018729$ [8],

$$\sqrt{M}_l = 1 + \frac{4(2l+1)^2-1}{16(2\eta\rho)^{1/2}} + \frac{[4(2l+1)^2-1](4(2l+1)^2-9]}{2[16(2\eta\rho)]^2} + \frac{[4(2l+1)^2-1][4(2l+1)^2-9][4(2l+1)^2-25]}{6[16(2\eta\rho)]^3}.$$

nucleon mass m= 931.5 MeV, square of electronic charge e^2=1.4398 MeV fm, $\hbar$= 197.329 Mev fm, and radial distance parameter $r_0 = \frac{R}{A_e^{1/3}+A_d^{1/3}}$ expressed in fm unit. The constant 'd' is l-dependent and it helps estimate the value of decay life-time of particles pushed out with angular momentum $l > 0$.

We use $R = 9.5$ fm and $P = 10^{-3}$ for all types of α-daughter nuclei. The radial distance R=9.5 fm is a distance over which the value of amplitude of the resonant wave function reduces to a small value of $\frac{1}{e}$.

We apply the formulation to several cases of α-daughter nuclei. The experimental data are explained quite well by our calculated results in the situations of l=0,1, 2, 3, 5, shown in Table 1.

TABLE I: Comparison of experimental values [6] of α-decay half-lives and results of present calculation obtained by using formula (5)

$^{A}_{Z}$	Q(MeV)	l	log$T_{1/2}^{expt}(s)$	log$T_{1/2}^{form}(s)$
$^{113}_{54}$	3.090	0	3.89	3.166
$^{151}_{64}$	2.652	0	15.03	14.80
$^{171}_{76}$	5.371	2	2.690	2.489
$^{175}_{80}$	7.060	2	-1.960	-2.347
$^{211}_{84}$	7.595	5	-0.280	-1.539
$^{237}_{94}$	5.748	1	12.120	9.337
$^{241}_{96}$	6.185	3	11.280	8.421
$^{255}_{102}$	8.442	5	4.200	2.574

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