# Searching for the conditions of convergence of statistical ensembles for fragmentation of finite nuclei

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Statistical models are extensively used to study the nuclear multifragmentation reactions at intermediate energies. In models of statistical disassembly of a nuclear system formed by the collision of two heavy ions one assumes that the hot and compressed nuclear system expands and subsequently fragments into composites of different masses depending upon the initial conditions. The fragmentation of the nucleus into available channels (depends on phase space) can be solved in canonical and grand canonical ensembles where the underlying physical assumption is fundamentally different. In principle the results from both ensembles agree only in the thermodynamical limit when the number of particles become infinite. The aim of this work to show the results from the canonical and the grand canonical models can agree even for finite nuclei under certain conditions and to search for these conditions.

In canonical model, the partitioning from the system with $N_0$ neutrons and $Z_0$ protons is done such that particle conservation in maintained in each channel. In this model the average number of composites with $N$ neutrons and $Z$ protons is given by

$$\langle n_{N,Z}\rangle_c \;=\; \omega_{N,Z}\frac{Q_{N_0-N,Z_0-Z}}{Q_{N_0,Z_0}} \qquad (1)$$

where $Q_{N_0,Z_0}$ is the total partition function and $\omega_{N,Z}$ is the partition function of the composite.

In the grand canonical ensemble, total number of particle in each channel is not conserved, only the average number of neutron and proton are conserved. In this case, the average number of composites with $N$ neutrons and $Z$ protons is given by [2]

$$\langle n_{N,Z}\rangle_{gc} \;=\; e^{\beta\mu_n N+\beta\mu_p Z}\omega_{N,Z} \qquad (2)$$

The neutron and proton chemical potentials $\mu_n$ and $\mu_p$ are determined by solving two equations $N_0 = \sum Ne^{\beta\mu_n N+\beta\mu_p Z}\omega_{N,Z}$ and $Z_0 = \sum Ze^{\beta\mu_n N+\beta\mu_p Z}\omega_{N,Z}$.

We compare the total charge distribution $\langle n_Z\rangle = \sum_N\langle n_{N,Z}\rangle$ obtained from both the ensembles at different source asymmetries $y = (N_0 - Z_0)/(N_0 + Z_0) = 0.33$, 0.17 and 0 at the fixed temperature 5 MeV, freeze-out volume at $3V_0$ and source size $A_0 = 60$. The difference in result is maximum at the highest asymmetry 0.33 where fragmentation is less and the disassembly of the nucleus results in more of 'liquid-like' fragments or higher mass fragments. As one decreases the asymmetry, fragmentation increases, the number of such

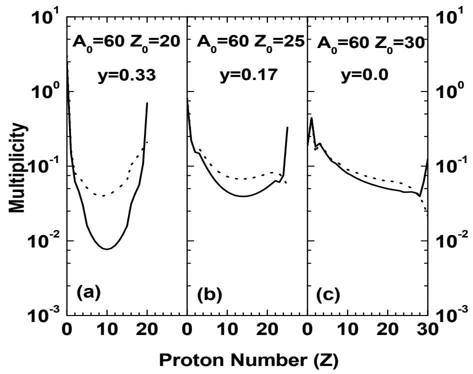


FIG. 1: Total charge distribution at $T = 5.0$ MeV and $V_f = 3V_0$ from canonical (solid lines) and grand canonical model (dotted lines) of the sources having same $A_0 = 60$ but different isospin asymmetry (a) $y = 0.33$, (b) 0.17 and (c) 0.

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higher mass fragments decrease (at the expense of the lower mass ones) and the results from the canonical and grand canonical ensembles begin to converge. This is easily seen at the two lower asymmetries. For the symmetric nucleus ($y = 0$), the results from both the ensembles are very close to each other since fragmentation is maximum for this case, the nucleons and the lower mass fragments dominating the distribution.

Similar effect is observed by increasing the temperature of a particular source ($Z_0 = 25, A_0 = 60$) at a fixed freeze-out volume $3V_0$ (Fig. 2). The difference in result between both the ensembles is maximum at the lowest temperature 3.8 MeV and the difference decreases with the increase of temperature.

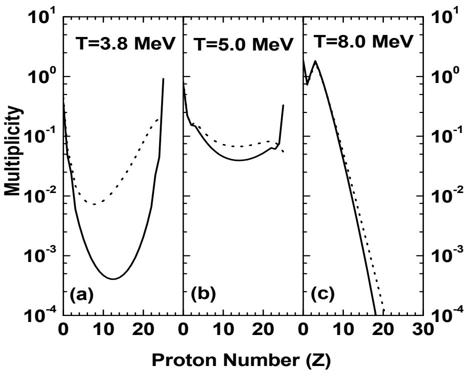


FIG. 2: Total charge distribution of $A_0 = 60$, $Z_0 = 25$ system from canonical (solid lines) and grand canonical model (dotted lines) at same $V_f = 3V_0$ but three different temperatures (a) 3.8 MeV , (b) 5 MeV and (c) 8 MeV.

In order to investigate the effect more, we have calculated the ratio (normalized) of higher mass fragments formed to that of the total number of fragments (total multiplicity). The fragment whose size is more than 0.8 times $A_0$ (more than 80% of the source in size) are considered as higher mass fragments i.e. the ratio is defined as

$$\eta = \frac{\sum_{A>0.8A_0}^{A_0} \langle n_{N,Z} \rangle}{\sum_{A=1}^{A_0} \langle n_{N,Z} \rangle} \quad (3)$$

In Fig 3.a we show the variation of this ratio as a function of asymmetry (keeping temperature, source size and freeze-out volume fixed) and it is seen that the ratio decreases with decrease in $y$. This shows that for a source with higher values of $y$, the fraction of higher mass fragments formed as a result of fragmentation is more as compared to those with lower $y$ values. Fig 3.b indicates the decrease of higher mass fragment production with the increase of temperature for the source ($Z_0 = 25, A_0 = 60$) at a fixed freeze-out volume $3V_0$.

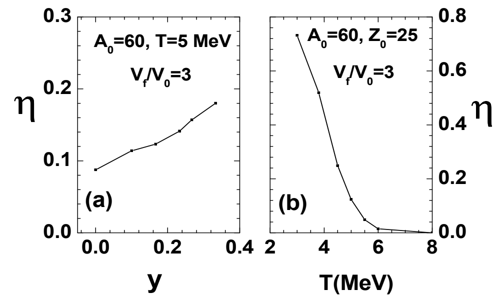


FIG. 3: Variation of $\eta$ with (a) isospin asymmetry and (b) temperature from grand canonical model.

We have shown that results from both models are in agreement for finite nuclei provided the nucleus fragments predominantly into nucleons and low mass clusters. This condition can be achieved either by decreasing the asymmetry of the source or by increasing the temperature. This convergence also can be achieved by increasing freeze-out volume of the fragmenting nucleus or by increasing the source size [3]. The main message that we wish to convey in this work is that while canonical and grand canonical models have very different underlying physical assumption, the results from both models can be in agreement with each other provided the contribution of higher mass fragments in nuclear disassembly is insignificant.

## References

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