Masses of di-mesonic molecular states

Ajay Kumar Rai,* Harsh R. Shah, and Aditi Toshniwal
Applied Physics Department, Sardar Vallabhbhai National Institute of Technology, Surat, Gujarat-395 007, INDIA

Introduction

The forces binding the quarks into mesons and baryons are described by Quantum Chromodynamics(QCD). But certain states like molecular states [1], tetraquark [2] and hybrid mesons [3] do not fit with such classifications of hadrons are been studied under framework of Bag Model [4] and non-relativistic potential models [5]. The non-relativistic potential models are successful at the heavy flavor sectors, while nonrelativistic harmonic oscillator potentials [6] and Bethe-Salpeter approach under harmonic confinement [7] are successful at low flavor sectors for their mass predictions. We calculated the binding energies and masses of exotic hadron resonances by treating them as di-mesonic molecules based on confinement scheme for mesons have been employed.

Theoretical framework

Based on the idea of finding approximations for the lowest energy eigenstates the variational method is used here. In this method we uses a gaussian type trial wave function having dependence over variational parameters. Here the di-mesonic Hamiltonian is,

$$\mathcal{H} = \sqrt{p^2 + m_{h_1}^2} + \sqrt{p^2 + m_{h_2}^2} + V(r) + V_{SD} \tag{1}$$

where m_{h_1} and m_{h_2} are masses of mesons, p is the relative momentum of the two mesons and V(r) is molecular interaction potential between two mesons.

We expressed it as the asymptotic expression($r \rightarrow \infty$) of the confined one gluon exchange interaction (COGEP) given by [9, 10]

TABLE I: Masses of mesons [8](in GeV)

Mesons	Mass(GeV)	Mesons	Mass(GeV)
π^+	0.140	D	1.865
K^+	0.494	D^*	2.007
K^0	0.498	B	5.279
K^*	0.892	B^*	5.325
η	0.548	B_s	5.366
ρ	0.775	B_s^*	5.415

$$V(r) = \frac{-k_{mol}}{r} e^{-C^2 r^2/2} \tag{2}$$

where k_{mol} is the residual strength of the strong interaction coupling and C is effective color screening parameter of the confined gluons. We added separately the spin-dependent part

$$V_{SD} = \frac{8}{9} \frac{k_{mol}}{m_{h_1} m_{h_2}} \vec{S_1} \cdot \vec{S_2} |\Psi|^2 \tag{3}$$

The raidal wave function for the ground state of di-meson system is assumed as [11, 12]

$$R(r) = \mu^{\frac{3}{2}} \left(\frac{2(n-1)!}{\Gamma(n+l+\frac{1}{2})} \right)^{\frac{1}{2}} (\mu r)^l \, e^{-\mu^2 r^2/2} L_{n-1}^{l+\frac{1}{2}}(\mu^2 r^2) \tag{4}$$

where μ is the variational parameter and $L_{n-1}^{l+\frac{1}{2}}(\mu^2 r^2)$ is Laguerre polynomial. The ground state energy is obtained by minimizing the expectation value of $\mathcal{H}$ as

$$E(\mu) = \langle R(r)|\mathcal{H}|R(r)\rangle, \qquad \frac{dE(\mu)}{d\mu} = 0 \tag{5}$$

We fix k_{mol}=0.45 and C=0.47 GeV for the calculation of all heavy-light and heavy-heavy combinations of mesons. For light sector

*Electronic address: raiajayk@gmail.com

TABLE II: Low lying masses of Multiquarks as di-mesonic molecule

System h_1-h_2	J^{PC}	μ GeV^2	BE GeV	Mass GeV	Expt. [8] GeV
$K^+ - K^+$	0^{++}	0.082	0.0042	0.9922	$f_0(0.980)$
$\pi^+ - K^*$	1^{+-}	0.084	0.0183	1.0568	$h_1(1.170)$
$\eta - \eta$	0^{++}	0.063	0.0028	1.0988	(1.092)[13]
$\rho - K^0$	1^{+-}	0.052	0.0019	1.2754	$K_1(1.273)$
$K^0 - K^*$	1^{+-}	0.048	0.0015	1.3919	$K_1(1.400)$
$\rho - \rho$	2^{++}	0.037	0.0010	1.5510	$f_0(1.590)$
$\rho - K^*$	2^{++}	0.034	0.0008	1.6678	$f_0(1.750)$
$K^* - K^*$	2^{++}	0.032	0.0006	1.7846	$f_1(1.950)$
$D^* - \pi^+$	1^{+-}	0.092	0.0309	2.1805	$D_1(2.420)$
$D - K^+$	0^{++}	0.310	0.0721	2.4311	(2.357)[13]
$D^* - K^0$	1^{+-}	0.311	0.0968	2.6019	$D_{sj}(2.573)$
$D - D^*$	1^{+-}	0.153	0.0045	3.8773	X(3.8715)
$D^* - D^*$	1^{++}	0.142	0.0051	4.0192	$\Psi(4.040)$
$B - K^+$	0^{++}	0.292	0.0489	5.8230	(5.727)[13]
$B^* - K^+$	1^{+-}	0.292	0.0488	5.8814	(5.770)[13]
$B_s - \eta$	0^{++}	0.320	0.0471	5.9651	(5.727)[13]
$D_s - B$	0^{++}	0.084	0.0011	7.2511	(7.244)[13]
$B - B$	0^{++}	0.042	0.0002	10.5602	(10.525)[13]
$B_s - B$	0^{++}	0.042	0.0002	10.6502	(10.680)[13]

the values of k_{mol}=0.68 and C=0.2 GeV are considered. We have used the experimental masses of the mesons for the present study (PDG)(Table-I) [8].

Conclusion and Discussion

Multiquark systems of di-mesonic states are studied in a nonrelativistic model. The masses of several di-mesonic states of light-light, heavy-light and heavy-heavy systems are computed using variational method. The inter mesonic interactions are taken due to the asymptotic expression of the confined one gluon echange interaction among the quarks [9]. While the mesonic masses have been taken from experiemntal results, the Van-der-Waal like interaction of di-mesonic binding energies have been computed using variational approach with the inter-mesonic interaction strength $k_{mol} = \alpha_s$, the strong running coupling constant at the respective hadronic scale.

Our predictions shown here are compared and identified with the experimentally known exotic hadronic states(see Table-II). The exotic states like $f_0(0.980)$, $f_0(1.590)$, $f_1(1.950)$, $D_1(2.420)$, $D_{sj}(2.573)$, X(3.8715), $\Psi(4.040)$ etc. are identified as di-mesonic states. These states are those whose spin-parity do not match with the expected quark anti-quark structure for mesons. We have parity and charge conjugation PC as +- for the combination of pseudoscalar-vector di-mesonic state, while for pseudoscalar dimesonic and vector-vector di-mesonic combinations PC value is ++. The candidates $f_0(0.980)$, $K_1(1.273)$, $D_{sj}(2.573)$ and (B-B)(10.525) are very close to our predicted masses of $(K^+ - K^+)$, $(\rho - K^0)$, $(D^* - K^0)$ and (B-B) respectively [8, 13].Here we made our attempt to understand the di-mesonic systems based on variational method. This study will be useful for the recently observed X,Y,Z exotic states.

Acknowledgment Dr A K Rai acknowledges the financial support from Department of Science & Technology, India under the fast Track project SR/FTP/PS-152/2012.

References

[1] N.A.Tornqvist Z.Phys. **C61**, 525 (1994).
[2] L.Mainani, F.Piccinini et al. Phys. Rev. D **71**, 014028 (2005).
[3] T.Barnes, F.E.Close et al. Phys. Rev. D **52**, 5242 (1995).
[4] R.L.Jaffe, Phys. Rev. D **38**, 195 (1977).
[5] J.Weinstein and N.Isgur Phys. Rev. D **27**, 588 (1983).
[6] N.Isgur and G.Karl Phys. Rev. D **18**, 4187 (1997).
[7] A.N. Mitra and I.L.Santhanam Z.Phys. **C8**, 33 (1981).
[8] K.Nakamura et al.(PDG) J. Phys. G. **37**, (2010).
[9] A.K.Rai, J.N.Pandya et al. Indian J. Phys. **80(4)**, 387 (2006)(and references therein).
[10] A.K.Rai. et al. Nucl. Phys. A **782**, 406 (2007).
[11] N.Devani and A.K.Rai Phys. Rev. D **84(7)**, 074030 (2011).
[12] N.Devani and A.K.Rai Eur. Phys. J. A **48**, 104 (2012).
[13] B.Silvestre-Brac, C.Semay., Z.Phys. C **57**, 273-282 (1993).