Light-cone representation of gravitational form factors in simulated hadron model

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Introduction

The light-cone Fock representation of composite systems such as hadrons in QCD has a number of remarkable properties. The generators of certain lorentz boosts are kinematical, knowing the wavefunction in one frame allows one to obtain it in any of other frame. Light-cone Fock state wavefunctions thus encode all of the bound state quark and gluon properties of hadrons including their spin and favor correlations in the form of universal process and frame-independent amplitudes. The light-cone wavefunctions also specify the multi quark and gluon corrrelations of the hadron. One can construct any spacelike electromagnetic, electroweak, or gravitational form factor from the diagonal overlap of the LC wavefunctions [1]. We present a simple self-consistent model of an effective composite spin-$\frac{1}{2}$ system based on the quantum fluctuations of the electron in QED. The model is developed after the structure which occurs in the one-loop Schwinger $\alpha/2\pi$ correction to the electron magnetic moment [1]. We represent a spin- $\frac{1}{2}$ system as a composite of a spin- $\frac{1}{2}$ fermion and spin-one vector boson with arbitrary masses. We compute the various contributions to the form factors of the energy-momentum tensor. We simulate the wavefunction by taking the derivative w.r.t M^2 in the denominator, so that wavefunction converges at the end point of x.

1. Gravitational form factors

The light-cone Fock representation allows one to compute all matrix elements of local currents as overlap integrals of the light-cone Fock wavefunctions. The local operators for the energy-momentum tensor $T^{\mu\nu}(x)$ and the angular momentum tensor $M^{\mu\nu\lambda}(x)$, one can directly compute momentum fractions, spin properties, the gravitomagnetic moment, and the form factors $A(q^2)$ and $B(q^2)$ appearing in the coupling of gravitons to composite systems. The form factors of the energy-momentum tensor for a spin-$\frac{1}{2}$ composite are defined by

$$\langle P'|T^{\mu\nu}(0)|P\rangle = \bar{u}(P')\Big[\,A(q^2)\gamma^{(\mu}\bar{P}^{\nu)} + B(q^2)\frac{i}{2M}\bar{P}^{(\mu}\sigma^{\nu)\alpha}q_\alpha + C(q^2)\frac{1}{M}(q^\mu q^\nu - g^{\mu\nu}q^2)\Big]\,u(P)\,, \qquad (1)$$

where $q^\mu = (P'-P)^\mu$, $\bar{P}^\mu = \frac{1}{2}(P'+P)^\mu$, $a^{(\mu}b^{\nu)} = \frac{1}{2}(a^\mu b^\nu + a^\nu b^\mu)$, and $u(P)$ is the spinor of the system. We can obtain the light-cone representation of the $A(q^2)$ and $B(q^2)$ form factors of the energy-tensor Eq. (1). By calculating the $++$ component of Eq. (1), we find

$$\langle P+q,\uparrow\left|\frac{T^{++}(0)}{2(P^+)^2}\right|P,\uparrow\rangle = A(q^2)\ , \qquad (2)$$

$$\langle P+q,\uparrow\left|\frac{T^{++}(0)}{2(P^+)^2}\right|P,\downarrow\rangle = -(q^1-\mathrm{i}q^2)\frac{B(q^2)}{2M}\ . \qquad (3)$$

2. Simulated Model Calculations

The individual contributions of the fermion and boson fields to the energy-momentum form factors in QED are given Ref. [2]. The results obtained in the simulated model for fermion and boson fields are obtained as

$$A_f(q^2) = \int dx(x^3(1-x)(1+x^2)(I_1+I_2+BI_3) + x^5(1-x)^3(M-\frac{m}{x})^2 I_3), \qquad (4)$$

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$$A_b(q^2) = \int dx(x^2(1-x)^2(1+x^2)(I_4 + I_5 + B'I_6) + 2x^4(1-x)^4(M - \frac{m}{x})^2 I_6), \quad (5)$$

$$B_f(q^2) = -4M \int dx\ x^5(1-x)^3(M - \frac{m}{x})I_3 \quad (6)$$

$$B_b(q^2) = 4M \int dx\ x^5(1-x)^4(M - \frac{m}{x})I_3 \quad (7)$$

where

$$I_1 = \pi \int_0^1 \frac{1-\alpha}{D^2} d\alpha, I_2 = \pi \int_0^1 \frac{\alpha}{D^2} d\alpha,$$
$$I_3 = \pi \int_0^1 \frac{\alpha(1-\alpha)}{D^3} d\alpha, D = \alpha(1-\alpha)$$
$$(1-x)^2 q_\perp^2 - M^2 x(1-x) + m^2(1-x) + \lambda^2 x,$$
$$B = 2M^2 x(1-x) - (1-x)^2 q_\perp^2 - 2m^2(1-x) - 2\lambda^2 x \quad (8)$$

Similarly

$$I_4 = \pi \int_0^1 \frac{1-\alpha}{D_1^2} d\alpha, I_5 = \pi \int_0^1 \frac{\alpha}{D_1^2} d\alpha,$$
$$I_6 = \pi \int_0^1 \frac{\alpha(1-\alpha)}{D_1^3} d\alpha, D_1 = \alpha(1-\alpha)x^2 q_\perp^2 - M^2 x(1-x) + m^2(1-x) + \lambda^2 x, B' = 2M^2 x(1-x) - x^2 q_\perp^2 - 2m^2(1-x) - 2\lambda^2 x. \quad (9)$$

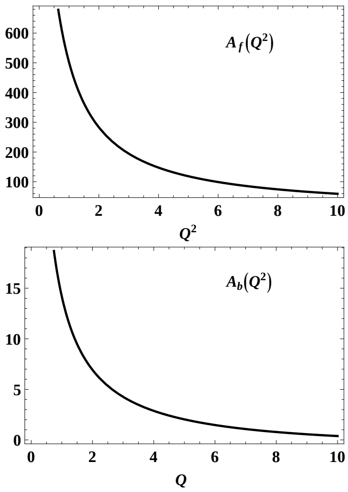

FIG. 1: Helicity non-flip gravitational form factors for the fluctuation of electron at one loop order in QED. The fermion mass $m_f = M$ and $\lambda = 0$.

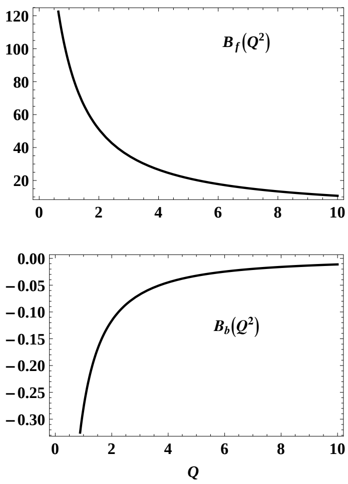

FIG. 2: Helicity flip gravitational form factors for the fluctuation of electron at one loop order in QED. The fermion mass $m_f = M$ and $\lambda = 0$.

3. Summary and Conclusions

The light-cone wavefunctions provide a general reprsentation of relativistic composite system. We calculate the gravitational form factor in simulated model. We can also simulate the hadron as a bound state of a quark and spin-0 diquark for further study of gravitational form factors.

4. Acknowledgments

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