Chaos in Strong Nuclear Interactions

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Introduction

Since last few decades, the issue of quark confinement is a challenging task which is yet to be solved completely. Many models of the field theory of quarks and gluons are proposed to understand the confinement problem and other closely related aspets [1–3]. Quantum chromodynamics (QCD) is quite successful theory to explain the features of the strong nuclear interactions in high energy regime. However, in low energy regime, it becomes very complicated. The chaotic field analysis of QCD has shown some interesting results to address the problems associated with confinement mechanism [4, 5]. In this work, we have tried to further explore the chaotic field dynamics of QCD in view of running nature of strong coupling constant (α_s).

Chaotic Field Analysis of QCD

The non-linearity present in the Yang-Mills equation makes the QCD lagrangian non-integrable [6]. This non-integrability cause chaos in quark-gluon system. Such non-integrability can be transcribed in an analogous way to the non-integrability in a non-central potential [7].

Let us consider a non-central potential potential in 2-dimensional space of the following form,

$$U(q_1, q_2) = \frac{1}{2} q_1^2 q_2^2 \,. \qquad (1)$$

The chaotic nature in potential Eq(1) can be traced back by considering the following general form of the Hamiltonian [8],

$$H = \frac{1}{2}(\dot{q_1}^2 + \dot{q_2}^2) + \frac{(1-\beta)}{12}(q_1^4 + q_2^4) + U, \qquad (2)$$

with $\beta = 1$, the Hamiltonian (2) reduces to the problem with the non-central potential given by Eq(1) while for $\beta = 0$ it is an integrable one. In order to make an analogy of non-central potential (1) with the non-Abelian gauge theory to study the non-linearity in strong nuclear interactions, the Yang-Mills equation (without quarks) for the simple choice of non-Abelian gauge group SU(2) can be written as follows,

$$\partial_\mu F^a_{\mu\nu} + g_s \epsilon^{abc} A^b_\mu F^c_{\mu\nu} = 0, \qquad (3)$$

where the gauge field strength $F^a_{\mu\nu}$ is given by,

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s \epsilon^{abc} A^b_\mu A^c_\nu, \qquad (4)$$

here, $a, b, c = 1, 2, 3$ and $\mu, \nu = 0, 1, 2, 3$. If the potential A^a_μ is chosen to be time dependent in the gauge $\partial_i A^a_i = 0$ and $A^a_0 = 0$, then the Eq(3) may be reduced to the form given below,

$$\ddot{B}^a_i + g_s^2 (A^b_j A^b_j A^a_i - A^a_j A^b_j A^b_i) = 0, \qquad (5)$$

where dots show time derivatives. If we consider the Hamiltonian given below,

$$H_A = \frac{1}{2}(\dot{A}^a_i)^2 + \pi\alpha_s \{(A^a_i A^a_i)^2 - (A^a_i A^a_j)^2\}, \qquad (6)$$

where, $\alpha_s = g_s^2/4\pi$ is the strong coupling constant. The equation of motion given by Eq(5) can also be derived from this Hamiltonian. The quantum spectrum of such systems may be described in terms of the energy eigenvalues ($E_1 \leq E_2 \leq E_3 ... E_n \leq ...$) of the potential discussed above. The distinction between regular and irregular spectral sequences

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may be analysed with the distribution of nearest neighbor spacing with the energy difference $\triangle E_n = E_{n+1} - E_n$ between $(n+1)$th and nth level [5]. The probability P(Y) of a given spacing $\triangle E$ (between neighboring levels in the neighborhood of energy level E) for such chaotic potential has been found to be very close to the Wigner distribution as given below [9],

$$P(Y) = \frac{\pi}{2} Y exp\left(-\frac{\pi Y^2}{4}\right), \qquad (7)$$

where, $Y = \triangle E / < \triangle E >$ being the mean level spacing. The running nature of the strong coupling constant (α_s) may thus be used to explore the regularities and irregularities at different energy (length) scales to explain the quark confinement and quark-gluon plasma (QGP) formation. The strong coupling constant (α_s) observed through deep inelastic scattering (DIS) experiments is ranged as, $0.1 < \alpha_s < 0.5$. The perturbative QCD domain is defined in range $0.1 < \alpha_s < 0.2$ and non-perturbative QCD domain in range $0.2 < \alpha_s < 0.5$. If we compare the role of α_s in Eq(6) with the β in Eq(2), it becomes clear that the strong coupling constant may become helpful in describing the chaotic view in different energy regimes of strong interactions.

Results and Conclusions

If one considers the extreme values of α_s in both the regimes (i.e., $\alpha_s \sim 0$ in perturbative sector and $\alpha_s \sim 1$ in non-perturbative sector), then it is evident from Eq(6) that the non-integrable potential is present only in the regime of non-perturbative sector. The chaos is, therefore, present when the quarks are confined while the absence of non-linearity, i.e., $\alpha_s \sim 0$, leads to the asymptotically free states (formation of QGP) perhaps having no chaos. Even the presence of sources (the dynamical quarks) in the present formulation may not influence the observations in terms of $P(Y)$ because the second term in Eq(6) remains intact. It is worth to notice that the complexity and universality of behavior were earlier thought to be non-compatible. But the chaos has showed the randomness and universality could coexist in different phenomena in nature as evident by the present analysis of strong nuclear interactions.

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