Radiative transitions of Charmonium and Bottomonium states in Relativistic phase spaces

Antony Prakash Monteiro[1], K. B.Vijaya Kumar[2]*, and A. P. Radhakrishna[1]
[1]*Department of Physics, St Philomena College, Darbe, Puttur, 574202, India*
[2]*Department of Physics, Mangalore University, Mangalagangothri, Mangalore, 574199, India*
*E-mail:kbvijayakumar@yahoo.com

Introduction

One of the most challenging fields in theoretical particle physics is heavy quark physics. The huge amount of data available on hadrons needs to be explained in order to explore this field. Meson radiative transitions deserve a lot of investigation since they are very easily produced and could help us understand theory of strong interactions. Since they belong to the non-perturbative regime of QCD they cannot be described from first principles. One of the theories which had number of successes in describing non perturbative part of QCD is the quark model. In this line, study of radiative transitions of some heavy meson systems such as charmonium and bottomonium states is of significance in testing various meson potentials and wave functions since the transition operator is very well known. In dealing with radiative transitions some typical approximations are usually in use. Most of these approximations are taken from atomic and nuclear physics where they describe radiative transitions rather well. But when applied to mesons they are not always justified.

Although these approximations are fully justified in atomic or nuclear physics, it is not obvious that they continue to work when applied to mesons. Indeed, in this sector the transition energy is typically $E\gamma = k\gamma = 0.1$–0.5 GeV, while the size of the source is roughly $R = 0.5$–1fm = 2.5–5GeV^{-1}, so that the long-wavelength condition $k\gamma R \ll 1$ is not really justified. Comparing the photon energy to the mass of the emitting meson also convinces us that a non-relativistic phase space is probably not appropriate.

The rate for transitions from a $^3S_1$ state to ^3P_J state [1] is given by,

$$\Gamma_{(^3S_1\rightarrow\gamma^3P_J)}=(2J+1)\frac{4}{27}e_q^2\alpha k_0^3|I_{PS}|^2,$$

where k_0 is the energy of the emitted photon, e_q is the charge of the quark, α is the fine structure constant and I_{PS} is the radial overlap integral which has the dimension of length.

$$I_{PS}=\langle P|r|S\rangle=\int_0^\infty r^3 R_P(r)R_S(r)dr$$

with $R_{S,P}(r)$ being the normalised radial wave functions for the corresponding states. The transition from ^3P_J levels to a $^3S_1$ level is described by the expression for the rate

$$\Gamma_{(^3P_J\rightarrow\gamma^3S_1)}=\frac{4}{9}e_q^2\alpha k_0^3|I_{SP}|^2$$

For transitions $^1P_1$ --> $^1S_0$ the same above expression is used to calculate the rate.

The allowed M1 transitions are essentially $^3S_1$ --> $^1S_0$ and $^1S_0$ --> $^3S_1$. The rate for transitions from a $^3S_1$ state to $^1S_0$ state is given by

$$\Gamma_{(n^3S_1\rightarrow\gamma m^1S_0)}=\frac{4}{3m^2}e_q^2\alpha k_0^3|I_{mn}|^2,$$

where I_{mn} is the overlap integral for unit operator between the coordinate wave functions of the initial and the final meson states and m is the mass of the quark.

$$I_{mn}=\int_0^\infty r^2 R_{nS}(r)R_{mS}(r)dr$$

For transitions from $^1S_0$ state to $^3S_1$ state the following expression for the rate is used

$$\Gamma_{(n^1S_0\rightarrow\gamma m^3S_1)}=\frac{4}{m^2}e_q^2\alpha k_0^3|I_{mn}|^2$$

The above radiative decay widths are calculated with relativistic phase spaces. The expressions are derived for the mesons having the same flavour wave functions ($c\bar{c}$ and $b\bar{b}$).

Results and discussions

We have calculated the E1 and M1 transition widths for charmonium and bottomonium states. These transitions are well-known and are reported in PDG [2]. We have studied the transitions allowed by long wavelength approximation using relativistic phase spaces. They are ^3P_J --> $^3S_1$ and $^3S_1$ --> ^3P_J (E1 transitions); $^3S_1$ --> $^1S_0$ and $^1S_0$ --> $^3S_1$ (M1 transitions). Also we have studied a particular E1 transition corresponding to the decay of $^1P_1$ --> $^1S_0$. In our calculations the experimental values of the meson masses have been used. Our results are shown in the tables 1-2.

Conclusions

Radiative decay widths have been investigated in non relativistic quark model for heavy mesons. The decay widths of most of the transitions agree with experimental values. The results obtained indicate that better results are obtained since relativistic phase space is used.

Table1 Radiative decay widths of bottomonium states

Transition	Expl. value Γ(keV)	Calculated Γ (keV)
$^3S_1$ -->^3P_J		
$\Upsilon(2S) \to \chi_{b0}(1P)\gamma$	1.22 ± 0.16	1.63
$\Upsilon(2S) \to \chi_{b1}(1P)\gamma$	2.21 ± 0.22	2.52
$\Upsilon(2S) \to \chi_{b2}(1P)\gamma$	2.29 ± 0.23	2.56
$\Upsilon(3S) \to \chi_{b0}(2P)\gamma$	1.2 ± 0.16	1.02
$\Upsilon(3S) \to \chi_{b1}(2P)\gamma$	2.56 ± 0.34	1.69
$\Upsilon(3S) \to \chi_{b2}(2P)\gamma$	2.66 ± 0.41	1.93
^3P_J--> $^3S_1$		
$\chi_{b0}(1P) \to \Upsilon(1S)\gamma$	seen	82.90
$\chi_{b1}(1P) \to \Upsilon(1S)\gamma$	seen	104.36
$\chi_{b2}(1P) \to \Upsilon(1S)\gamma$	seen	118.94
$\chi_{b0}(2P) \to \Upsilon(2S)\gamma$	seen	17.56
$\chi_{b1}(2P) \to \Upsilon(2S)\gamma$	seen	23.88
$\chi_{b2}(2P) \to \Upsilon(2S)\gamma$	seen	28.03

Table 2 Radiative decay widths of charmonium states

Transition	Expl. value Γ(keV)	calculated Γ (keV)
	$^3S_1$ --> ^3P_J	
Ψ(2S) --> χ_{c0}(1P)γ	25.76 ± 3.81	25.42
Ψ(2S) --> χ_{c1}(1P)γ	24.10 ± 3.49	22.35
Ψ(2S) --> χ_{c2}(1P)γ	21.61 ± 3.28	15.49
	^3P_J --> $^3S_1$	
χ_{c0}(1P) --> J/ Ψ(1S) γ	92.40 ± 41.52	146.21
χ_{c1}(1P) --> J/ Ψ(1S) γ	240.24 ± 40.73	301.72
χ_{c2}(1P) --> J/ Ψ(1S) γ	270.00 ± 32.78	401.14
	$^1P_1$ --> $^1S_0$	
h_c--> η_c(1S) γ	seen	618.98
	$^3S_1$ --> $^1S_0$	
J/ Ψ(1S) --> η_c(1S) γ	1.13 ± 0.35	2.47

References

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