Electromagnetic transition widths in B_c meson

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Introduction

An electromagnetic transition between quarkonium states, occurring via emission of a photon, leaves clear experimental signature of a monochromatic photon. This provides a useful production mechanism for discovery and study of the lower-lying states, and a unique window on the dynamics of such systems. The M1 dipole radiative transition widths can be useful for the right identification of the newly observed states[1].

Theretical formulation

The radiative widths are calculated in the dipole approximation. The E1 dipole transition width is given by[2]

$$\Gamma_{E1} = \frac{4\alpha}{9}\left(\frac{e_Q m_{\bar q} - e_{\bar q} m_Q}{m_{\bar q}+m_Q}\right)^2 k^3 \left|\langle f\left|r\right|i\rangle\right|^2 \times \frac{E_f}{M_i}\mathcal{S}_{if} \qquad (1)$$

Here, α is the fine structure constant, $k = (M_i^2 - M_f^2)/2M_i$ is the photon energy, $e_{\bar q}$ and e_Q are the quark charges in units of the proton charge, E_f is the energy of the final meson state, M_i is the mass of the initial meson state, and $m_{\bar q}$ and m_Q are the quark masses. Also

$$\langle f\left|r\right|i\rangle = \int dr R_i(r) r R_f(r). \qquad (2)$$

The M1 dipole transition width $(\Gamma\left(i \to f+\gamma\right))$ between S-wave levels is given by[2]

$$\Gamma_{M1} = \frac{16\alpha}{3}\mu_M^2 k^3\left(2J_f+1\right)\left|\langle f\left|j_0\left(kr/2\right)\right|i\rangle\right|; \qquad (3)$$

where α is the fine structure constant, k is the photon energy; M_i and M_f are initial and final state masses respectively, J_f is the angular momentum of the final state, and the magnetic dipole moment

$$\mu_M = \frac{m_{\bar q} e_Q - m_Q e_{\bar q}}{4 m_Q m_{\bar q}}; \qquad (4)$$

m and e are quark/anti-quark mass and charge respectively. The overlap integral is

$$\langle f\left|j_0(kr/2)\right|i\rangle = \int dr R_i(r) j_0(kr/2) R_f(r). \qquad (5)$$

To obtain the masses of the B_c meson Potential model framework is used. We employ the hamiltonian[3–6]

$$H = \sqrt{\mathbf{p}^2 + m_Q^2} + \sqrt{\mathbf{p}^2 + m_{\bar Q}^2} + V(r); \qquad (6)$$

$$V(r) = -\frac{4\alpha_S}{3r} + Ar + V_0, \qquad (7)$$

here α_S is the running strong coupling constant, A is a potential parameter and V_0 is a constant. The potential chosen consists of two parts. The first part is a contribution due to the one gluon exchange potential and represents asymptotic freedom while the second part takes care of the confinement of the quarks.

In order to take relativistic corrections into account we expand the hamiltonian upto order $\mathbf{p}^6$. The value of the QCD coupling constant α_S is determined through[7]

$$\alpha_S\left(M^2\right) = \frac{4\pi}{\left(11 - \frac{2}{3}n_f\right)\ln\frac{M^2+M_B^2}{\Lambda^2}}, \qquad (8)$$

where $M = 2m_Q m_{\bar q}/\left(m_Q + m_{\bar q}\right)$, $M_B = 0.95$ GeV and $\Lambda = 413$ MeV. The masses employed

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TABLE I: M1 transition widths of heavy quarkonia.

Transition	M_i (GeV)	M_f (GeV)	k (GeV)	Γ(keV)				
				This work	Ref.[8]	Ref. [9]	Ref. [10]	Ref. [11]
$1^3P_2 \rightarrow 1^3S_1\gamma$	6.772	6.334	0.423	77.88	83	107	102.9	112.6
$1P_1' \rightarrow 1^3S_1\gamma$	6.757	6.334	0.409	8.68	11	13.6	8.1	0.1
$1P_1' \rightarrow 1^1S_0\gamma$	6.757	6.269	0.470	93.27	80	132	131.1	56.4
$1P_1 \rightarrow 1^3S_1\gamma$	6.751	6.334	0.404	59.35	60	78.9	77.8	99.5
$1P_1 \rightarrow 1^1S_0\gamma$	6.751	6.269	0.465	12.71	13	18.4	11.6	0.0
$1^3P_0 \rightarrow 1^3S_1\gamma$	6.716	6.269	0.370	52.23	55	67.2	65.3	79.2
$1^3S_1 \rightarrow 1^1S_0\gamma$	6.334	6.269	0.065	0.086	0.08	0.033	0.060	0.134
$2^3S_1 \rightarrow 2^1S_0\gamma$	6.879	6.865	0.014	0.001	0.010	0.017	0.010	0.029
$2^3S_1 \rightarrow 1^1S_0\gamma$	6.879	6.269	0.582	0.521	0.6	0.428	0.098	0.123

in the present calculation are $m_c = 1.55$ GeV and $m_b = 4.88$ GeV.

We use variational method in which the trial wave function is a Gaussian given by

$$R_{nl}(\mu, r) \; = \; \mu^{3/2}\left(\frac{2\,(n-1)!}{\Gamma\,(n+l+1/2)}\right)^{\frac{1}{2}}(\mu r)^l \times \\ e^{-\mu^2r^2/2}L_{n-1}^{l+1/2}(\mu^2r^2); \qquad (9)$$

μ is a variational parameter and L is Laguerre polynomial. We set the value of $A = 0.175$ GeV^2. The variational parameter μ is determined by making use of the virial theorem[12]. With this value of μ spin-averaged mass of heavy quarkonia is obtained by

$$H\psi = E\psi \qquad (10)$$

The SA mass is matched with the experimental SA mass in order to determine the constant V_0. The vaue of $V_0 = -0.282$ GeV.

We add separately (in Eq.10) the spin-dependent part to the usual one gluon exchange potential between the quark antiquark for computing the hyperfine and spin-orbit shifting of the low-lying S wave states. The form of the spin-dependent potential can be found in ref [10].

Results and discussion

The E1 and M1 transition widths obtained within the present scheme are tabulated in Table (I). It is observed that for $b\bar{c}$ the widths of the E1 and M1 transitions are in satisfactory agreement with other theoretical predictions. However, experimental measurements will be required to single out a particular approach.

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