# Relative importance of various Dirac structures in hadron-quark vertex for studies on leptonic decays of vector mesons

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**Introduction:** Meson decay studies has become a hot topic in recent years. Flavourless vector mesons play an important role in hadron physics due to their direct coupling to photons and thus provide an invaluable insight into the phenomenology of electromagnetic couplings to hadrons. Thus, a realistic description of vector mesons at the quark level of compositeness would be an important element in our understanding of hadron dynamics and reaction processes at scales where QCD degrees of freedom are relevant. There have been a number of studies on processes involving strong, radiative and leptonic decays of vector mesons. Such studies offer a direct probe of hadron structure and help in revealing some aspects of the underlying quark-gluon dynamics. In this work we study electromagnetic decays of ground state equal mass vector mesons: $\rho, \omega, \varphi, \psi$ and Y (each comprising of equal mass quarks) through the process, $V \rightarrow \gamma * \rightarrow e^- + e^+$ which proceeds through the coupling of quark-anti quark loop to the electromagnetic current in the framework of Bethe-Salpeter Equation (BSE), which is a conventional non-perturbative approach in dealing with relativistic bound state problems in QCD and is firmly established in the framework of Field Theory. From the solutions, we obtain useful information about the inner structure of hadrons which is also crucial in high energy hadronic scattering and production processes. We get useful insight about the treatment of various processes using BSE due to the unambiguous definition of the 4D BS wave function which provides exact effective coupling vertex (Hadron-quark vertex) of the hadron with all its constituents (quarks).

**Power counting rule:** In these studies, the main ingredient is the 4D hadron-quark vertex function Γ which plays the role of an exact effective coupling vertex of the hadron with all its constituents (quarks). The complete 4D BS wave function $\psi(P,q)$ for a hadron of momentum P and internal momentum q comprises of the two quark propagators (corresponding to two constituent quarks) sandwiching the hadron-quark vertex Γ. This 4D BS wave function is considered to sum up all the non-perturbative QCD effects in the hadron. Now, one of the main ingredients in 4D BS wave function (BSW) is its Dirac structure. The copius Dirac structure of BSW was already studied by C.H.L. Smith [1] much earlier. Recent studies [2] have revealed that various mesons have many different Dirac structures in their BS wave functions, whose inclusion is necessary to obtain quantitatively accurate observables. It was further noticed that all structures do not contribute equally for calculation of various meson observables.

Towards this end, to ensure a systematic procedure of incorporating various Dirac covariants from their complete set in the BSWs of various hadrons (pseudoscalar, vector etc.), we developed a naive power counting rule in ref.[3], by which we incorporate various Dirac structures in BSW, order-by-order in powers of inverse of meson mass, M, and thus naturally giving us a criterion so as to systematically choose among various Dirac structures from their complete set to write wave functions for different mesons. Using this power counting rule we can write the hadron-quark vertex function for a vector meson $\Gamma(\hat{q})$ as a polynomial in powers of $1/M$ (see Ref.[4] for details) up to order NLO as:

$$\Gamma^V(\hat{q}) = \Omega^V \frac{1}{2\pi i} N_V D(\hat{q}) \varphi(\hat{q}); \quad (1)$$

$$\Omega^V = \Big[i\gamma.\varepsilon A_0 + (\gamma.\epsilon)(\gamma.P)\frac{A_1}{M} + [q.\varepsilon - (\gamma.\epsilon)(\gamma.q)]\frac{A_2}{M} + [(\gamma.\varepsilon)(\gamma.P)(\gamma.q) - (\gamma.\varepsilon)(\gamma.q)(\gamma.P) + 2i(q.\varepsilon)(\gamma.P)]\frac{A_3}{M^2} + (q.\varepsilon)\frac{A_4}{M} + i(q.\varepsilon)(\gamma.P)\frac{A_5}{M^2}\Big]$$

Here, $N_V$ is the 4D BS wave function for a ground state vector meson with total momentum P and internal momentum q, and is worked out in the framework of Covariant Instantaneous Ansatz (CIA) to give explicit covariance to the full fledged 4D BS wave function $\psi(P,q)$ and hence to the Hadron-quark vertex function $\Gamma^V(\hat{q})$, employed for calculation of decay constants through 4D loop integrals. $D(\hat{q})$ is the universal denominator function which plays an important role in bringing out an exact interconnection between 3D and

4D BSE and hence in identifying the 4D hadron-quark vertex, while $\varphi(\hat{q})$ is the 3D BS wave function and appears as the solution of 3D BSE and takes a gaussian form, $\varphi(\hat{q}) = Exp.[-\frac{\hat{q}^2}{2\beta^2}]$, with β being the inverse range parameter. Here $\hat{q}$ is the component of internal hadron momentum orthogonal to the total hadron momentum P. Here, $A_i$ are the six dimensional and constant coefficients to be determined. Since we take constituent quark masses where the quark mass m is approximately half the hadron mass M, we can use the ansatz, q<< P ~ M [3-6] in the rest frame of the hadron. Then we can see that each of the 6 terms in the above equation receives suppression by powers of O(1/M), implying that the maximum contribution to calculation of any meson observable should come from leading order (LO) Dirac structures: $i\gamma.\varepsilon$ and $(\gamma.\epsilon)(\gamma.P)/M$, followed by next-to-leading order (NLO) Dirac structures: $[q.\varepsilon - (\gamma.\epsilon)(\gamma.q)]/M$ , $[(\gamma.\varepsilon)(\gamma.P)(\gamma.q) - (\gamma.\varepsilon)(\gamma.q)(\gamma.P) + 2i(q.\varepsilon)(\gamma.P)]/M^2$ , $(q.\varepsilon)/M$ and $i(q.\varepsilon)(\gamma.P)/M^2$.

**Leptonic decay constants of vector mesons:** Leptonic decay constants $f_V$ can be evaluated through the quark-loop diagram which gives the coupling of two-quark loop to the electromagnetic current and can be expressed as a loop integral [4]

$$f_V\epsilon_\mu(P) = \frac{\sqrt{3}}{M}\int d^4\, qTr[\psi_V(P,q)i\gamma_\mu], \quad (2)$$

$\varepsilon_\mu(P)$ being the polarization vector of a vector meson of momentum P satisfying $\epsilon.P = 0$. We make use of the six LO and NLO Dirac covariants in the BS wave function to calculate $f_V$ and in the process study the relevance of all these covariants to this calculation. The decay constant with incorporation of both LO and NLO Dirac covariants is expressed as: $f_V = f_V^0 + f_V^1 + f_V^2 + f_V^3 + f_V^4 + f_V^5$ where $f_V^0$ and $f_V^1$ are the contributions to $f_V$ from leading order (LO) Dirac structures : $i\gamma.\varepsilon$ and $(\gamma.\epsilon)(\gamma.P)/M$ , while $f_V^2,\ldots,f_V^5$ are the contributions from next-to-leading order (NLO) Dirac structures. For details of expression for $f_V$ see Ref.[4]. We note here that $f_V^2$ and $f_V^5$ are zero on account of equal mass kinematics. Using the method of least square fitting of data we find that the values of coefficients (with average error with respect to experimental data of 4%) should respectively be: $A_0 = 1$, $A_1 = .007 \pm .001$, $A_2 = 1.240 \pm .001$, $A_3 = -.415 \pm .001$, $A_4$=.014±.001 and $A_5 = -1.842 \pm .001$ to give the decay constant values of $\rho,\omega,\varphi,\psi$ and Y mesons as given in Table 1. These findings are completely in accordance with our power counting rule according to which the LO covariants in BS wave function should contribute maximum to $f_V$ followed by the next-to-leading order (NLO) covariants. And among the two LO covariants, the most leading covariant $i\gamma.\varepsilon$ contributes maximum to all vector mesons from ρ to Y. The drop in contributions to decay constants from NLO covariants vis-à-vis LO covariants is more pronounced for heavy mesons ψ and Y. And though LO and NLO Dirac covariants are sufficient to correctly predict amplitudes for φ and ψ vector mesons, but only LO covariants are sufficient for Y meson. But for ρ and ω mesons, the LO and NLO Dirac covariants are not sufficient to predict accurately their amplitudes and is thus necessary to include even higher order NNLO Dirac covariants [4] in their hadron-quark vertex functions. This can also be seen from Fig. 3 of Ref.[4]. These results on vector mesons are in complete conformity with the corresponding results on decay constants $f_P$ for pseudoscalar mesons $\pi, K, D, D_S$ and B [6] done recently. The numerical results for $f_V$, $f_P$ as well as $F_P$ (in [6]) obtained in our framework with use of LO and NLO covariants demonstrates the validity of our power counting rule which also provides a practical means of incorporating various Dirac covariants in BS wave function for a hadron. By this rule we get to understand relative importance of various covariants to calculation of meson observables and thus helps in improving our understanding of hadron structures.

Table 1: Decay constant $f_V$ values **(in MeV)** for mesons in BSE (in last column along with the corresponding experimental data) with individual contributions from various Dirac covariants along with the contributions from LO and NLO covariants and also their % contributions.

| | $f_V^0$ | $f_V^1$ | $f_V^3$ | $f_V^4$ | $f_V^{LO}$ | $f_V^{NLO}$ | $f_V^{LO}$% | $f_V^{NLO}$% | $f_V = f_V^{LO} + f_V^{NLO}$ |
|---|---|---|---|---|---|---|---|---|---|
| ρ | 115.60 | -0.68 | 93.36 | -0.26 | 114.90 | 93.10 | 55% | 45% | 208 (**Exp.=220.1±.9**) |
| ω | 115.50 | -0.69 | 93.00 | -0.25 | 114.80 | 92.70 | 56% | 44% | 207.5 (**Exp.=195±3.0**) |
| φ | 146.10 | -1.04 | 86.00 | -0.29 | 145.00 | 85.90 | 63% | 37% | 230.2 (**Exp.=228±3.0**) |
| ψ | 352.00 | -3.21 | 62.57 | -0.25 | 348.70 | 62.32 | 85% | 15% | 411 (**Exp.=410±3.0**) |
| Y | 661.70 | -6.28 | 52.68 | -0.22 | 655.55 | 52.50 | 92.6% | 7.4% | 707 (**Exp.=708±5.0**) |

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