Masses and radiative leptonic decay properties of Bc meson

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Introduction

We employ non-relativistic treatment with the help of Schrödinger equation in order to study the Bc spectroscopy. The Schrödinger equation for the bound state of Bc system is solved using numerical integration together with convexity arguments and nodal theorem for wave function [1]. The pure-leptonic decays of heavy mesons are very interesting from theoretical as well as experimental point of view [2,3]. In the present paper we have studied the decay constants and the leptonic decay width using the non-relativistic treatment.

Methodology

We solve the Schrödinger Equation numerically with the quark-antiquark potential of the form [4-6],

$$V(r) = -\frac{k\alpha_s}{r} + Ar^{\upsilon} + V_{SD} \tag{1}$$

Where A is the potential parameter, υ is a general potential index corresponding to the confining part of the potential. For present computation of masses and decay properties, we have taken as $\upsilon = 1$. α_s is the strong running coupling coefficient which can be determined from

$$\alpha_s(M^2) = \frac{4\pi}{\left(11-\frac{2}{3}n_f\right)\ln\frac{M^2+M_b^2}{\Lambda^2}} \tag{2}$$

Where the scale is taken as M=$2m_Q m_{\bar{q}}/(m_Q + m_{\bar{q}})$, M_b=0.95 GeV, Λ=413 MeV. We fit the values of k and A for ground state of $b\bar{b}$ using experimental value of b quark mass and then determine the c quark mass by fitting $c\bar{c}$ ground state mass [7]. We choose the scale for the Bc system as α_s = 0.255. The obtained values for k = 1.173, A = 0.17, m_b = 4.66 GeV and m_c = 1.275 GeV are employed for further computation. V_{SD} is the spin dependent part of the potential [8].

$$V_{SD} = V_{SS(r)}\left[S(S+1) - \frac{3}{2}\right] + V_{LS}(r)(\vec{L}.\vec{S}) + V_T(r)\left[S(S+1) - \frac{3(\vec{S}.\vec{r})(\vec{S}.\vec{r})}{r^2}\right] \tag{3}$$

The spin-orbit term containing $V_{LS}(r)$ and the tensor term $V_T(r)$ describe the fine structure of the meson state, while the spin-spin term containing $V_{SS}(r)$ proportional to $2(\overrightarrow{S_q}.\overrightarrow{S_{\bar{q}}})=S(S+1)-3/2$. The coefficient of these spin-dependent terms of Eq.3 can be written in terms of the vector and scalar parts of the static potential as [8]

$$V_{LS}(r) = \frac{1}{2m_1m_2r}\left(3\frac{dV_V}{dr} - \frac{dV_S}{dr}\right)$$

$$V_T(r) = \frac{1}{6\,m_1m_2r}\left(3\frac{d^2V_V}{dr^2} - \frac{1}{r}\frac{dV_S}{dr}\right) \tag{5}$$

$$V_{SS}(r) = \frac{1}{3m_1m_2}\nabla^2 V_V$$

The Bc mass spectroscopy is computed with these parameters and the result is given in Table 1.

TABLE I: Masses of Bc Meson (GeV)

State $n\,^{2S+1}L_J$	Present	[9]	[10]	[11]
$1\,^1S_0$	6.293	6.270	6.349	6.264
$1\,^3S_1$	6.317	6.332	6.373	6.337
$2\,^1S_0$	6.777	6.835	6.821	6.856
$2\,^3S_1$	6.811	6.881	6.855	6.899
$3\,^1S_0$	7.152	7.193	7.125	7.244
$3\,^3S_1$	7.187	7.235	7.210	7.280

Decay Constants

The pseudoscalar and vector decay constants are computed using the Van Royen Weisskopf formula for color are zero separation between the constituent quarks in ground state [12]

$$f_p = \sqrt{\frac{3}{\pi M_p}}\,R_{1S}(0)\ ;\ f_v = \sqrt{\frac{3}{\pi M_v}}\,R_{1S}(0) \tag{5}$$

M_p and M_v are the masses of the pseudoscalar and vector meson respectively. The values of f_p and f_V are given in the Table II with the charge radii of the S-wave Bc mesons.

TABLE II: pseudoscalar and vector decay constants

State	Decay constant (MeV)				
	Present	[14]	[16]	[17]	[18]
$1\,^1S_0$	412	350	360	456	607
$1\,^3S_1$	411	--	--	--	604

Radiative Leptonic Decay Width

In this section, we compute the radiative decay width using the relation [13]

$$\Gamma(Bc \to l\overline{\gamma\upsilon}) = \frac{G_F^2|V_{cb}|^2}{8\pi^2}\, f_{Bc}^2 m_{Bc}^3 \,\frac{m_l^2}{m_{Bc}^2}\,\left(1-\frac{m_l^2}{m_{Bc}^2}\right)^2$$

As the mass of the lepton is very low compared to the *Bc* meson, the decays of pseudoscalar mesons into light lepton pairs are helicity suppressed, i.e. their decay widths are suppressed m_l^2/m_{Bc}^2, therefore the above formula becomes

$$\Gamma(Bc \to \gamma l\bar{\upsilon}) = \frac{\alpha G_F^2|V_{cb}|^2}{2592\pi^2}\, f_{Bc}^2 m_{Bc}^3[x_b + x_c] \quad (6)$$

Where $\alpha = 1/137$ is the electromagnetic coupling constant, G_F Fermi coupling constant = 1.66×10^{-5}, $|V_{cb}|$=0.044 [13] is the CKM matrix element. x_b and x_c is given by:

$$x_b = \left(3 - \frac{m_{Bc}}{m_b}\right)^2 \text{ and } \quad x_c = \left(3 - 2\,\frac{m_{Bc}}{m_b}\right)^2 \qquad (7)$$

The computed radiative leptonic decay width for S-wave *Bc* mesons is 1.59×10^{-17} GeV which is comparable in order with 6.44×10^{-17} GeV as obtained by C. Cheng et al. [16].

Summary

The results from Table (I) suggests that our results for the *Bc* meson S-wave masses are in good agreement with that by D. Ebert et al [9] with small deviation from other references. It is also evident from Table II that the decay constants are in tune with other theoretical models too. As the experimental results for the same are not available, we compare the outcome of the present work with existing phenomenological models. It is found that the present non-relativistic computation can provide good framework to study *Bc* meson as both the quarks can be treated non-relativistically. Further study on the decay properties using fine-tuned parameters is underway.

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Reference

[1] Wolfgung LUCHA, Franz F. SCHOBERL, arXiv: hep-ph/9811453v2 9 Jan 1999, HEPHY-PUB 703/98 UWThPh-1998-58 October 1998
[2] C.-H. Chang and Y.-Q. Chen, Phys. Rev. D49 (1994) 3399.
[3] D.-S. Du, H.-Y. Jin and Y.-D. Yang, preprint BIHEP-Th/97-007, hep-ph/9705261
[4] A. K. Rai, R. H. Parmar, and P. C. Vinodkumar, J. Phys. G: Nucl. Part. Phys. 28, 2275 (2002),
[5] A. K. Rai, J. N. Pandya, and P. C. Vinodkumar, J. Phys. G: Nucl. Part. Phys. 31, 1453 (2005),
[6] A. K. Rai, B. Patel, and P. C. Vinodkumar, Phys. Rev. C 78, 055202 (2008).
[7] Particle Data Group, J. Beringer et al. (PR D86, 010001 (2012).
[8] Voloshin M B, Prog. Part. Nucl. Phys. 61, 455 (2008);arXiv:hep-ph/0711.4556v3.
[9] D Ebert, R N Faustov and V O Galkin, *Phys. Rev.* D67, 014027 (2003)
[10] A.K.Rai, P.C.Vinodkumar *Pramana J. Phys.*, Vol. 66, No. 5, May 2006
[11] V. O. Galkin, A. Yu. Mishurov and R. N. Faustov, Yad. Fiz. 55, 2175 (1992) [Sov. J. Nucl. Phys. 55, 1207 (1992)].
[12] E J Eichten and C Quigg, *Phys. Rev.* D49, 5845 (1994)
[13] Chao-Hsi Chang, Jian-Ping Cheng and Cai-Dian L¨u arXiv:hep-ph/9712325v2 21 Jan 1998, DESY 97-246 AS-ITP-97-027 December, 1997
[14] T.M. Aliev, M.Savci, Phys. Letts. B434 (1998) 358, hep-ph/9804407.
[15] C.-H. Chang, J-P Cheng and C-D L¨u, Phys. Letts. B425 (1998) 166, hep-ph/9712325.
[16] Chao-hsi Chang, Cai-Dian Lu, Guo-Li Wang, Hong-Shi Zong, Phys. Rev. D 60 (1999) 114013
[17] S S Gershtein, V V Kiselev, A K Likhoded and A V Tkabladze, *Phys. Rev.* **D51** (1995) 3613
[18] J N Pandya and P C Vinodkumar, Pramana J. Phys. 57(4) (2001) 821