Mass Spectra of D, D_s Mesons using Dirac formalism with Martin-like confinement potential

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Introduction

Remarkable progress at the experimental side, with various high energy machines such as BaBar, BELLE, B-factories, Tevatron, ARGUS collaborations, CLEO, CDF, DØ etc., for the study of hadrons has opened up new challenges in the theoretical understanding of light-heavy flavour hadrons. The existing results on excited heavy-light mesons are partially inconclusive, and even contradictory in several cases. The predictions of masses of heavy-light system for low-lying 1S and $1P_J$ states of D and D_s mesons were known from experiment [1] and few from the theory [2–4]. Here we study the mass spectra of D and D_s mesons in a relativistic framework.

Theoretical Framework

The bound constituent quark and antiquark inside the meson are in definite energy states having no definite momenta. However one can find out the momentum distribution amplitude for the constituent quark and antiquark inside the meson immediately before their annihilation to a lepton pair. Though the colour confinement of quarks are understood in terms of multigluon exchanging at the non-perturbative regime of the hadronic size, it is not feasible to compute theoretically from the QCD first principles. Thus one assumes various confinement mechanism to study the hadronic properties. In the present study, we assume that the constituent quarks in a meson core is independently confined by an average Martin-like potential of the form [5]

$$V(r) = \frac{1}{2}(1+\gamma_0)(\lambda r^{0.1} + V_0) \qquad (1)$$

To a first approximation, the confining part of the interaction is believed to provide the zeroth-order quark dynamics inside the meson core through the quark Lagrangian density

$$\mathcal{L}_q^0(x) = \bar{\psi}_q(x)\left[\frac{i}{2}\gamma^\mu \overleftrightarrow{\partial_\mu} - V(r) - m_q\right]\psi_q(x). \qquad (2)$$

The normalized quark wave functions $\psi(\vec{r})$ obtained from Eqn (2) satisfies the Dirac equation given by

$$[\gamma^0 E_q - \vec{\gamma}.\vec{P} - m_q - V(r)]\psi_q(\vec{r}) = 0. \qquad (3)$$

TABLE I: S-wave D ($c\bar{q}$, q = d,u) spectrum (in MeV).

nL	State	Present	Experiment Meson	Mass [1]	[6]	[7]
1S	1^3S_1	2013.3	$D^*(2010)$	2010.28±0.13	-	2010
	1^1S_0	1874.0		1869.62±0.15	-	1871
2S	2^3S_1	2581.0	$D^*(2600)$	2608.7 [8]	2639	2632
	2^1S_0	2501.7	$D(2550)$	2539.4 [8]	2567	2581
3S	3^3S_1	3088.9		-	3125	3096
	3^1S_0	3031.5		-	3065	3062
4S	4^3S_1	3567.8		-	-	3482
	4^1S_0	3521.6		-	-	3452
1P	1^3P_2	2455.1	$D_2^*(2460)$	2462.6 ± 0.7	-	2460
	1^3P_1	2348.0		-	-	2469
	1^3P_0	2276.6	$D_0^*(2400)$	2318 ± 29	-	2406
	1^1P_1	2317.3	$D_1(2420)$	2421.3 ± 0.6	-	2426
2P	2^3P_2	2907.0		-	2965	3012
	2^3P_1	2834.4		-	2960	3021
	2^3P_0	2786.0		-	2880	2919
	2^1P_1	2812.3		-	2940	2932

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The two component solution of Dirac equation can be written as

$$\psi_{nlj}(r) = \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} \qquad (4)$$

where the positive and negative energy solutions are written as

$$\psi_A^{(+)}(\vec{r}) = N_{nlj} \begin{pmatrix} \frac{ig(r)}{r} \\ \frac{(\sigma.\hat{r})f(r)}{r} \end{pmatrix} \mathcal{Y}_{ljm}(\hat{r}) \qquad (5)$$

$$\psi_B^{(-)}(\vec{r}) = N_{nlj} \begin{pmatrix} \frac{i(\sigma.\hat{r})f(r)}{r} \\ \frac{g(r)}{r} \end{pmatrix} (-1)^{j+m_j-l}\mathcal{Y}_{ljm}(\hat{r}) \qquad (6)$$

and N_{nlj} is the overall normalization constant. The radial solutions f(r) and g(r) is obtained numerically to yield the energy eigen values. The parameters are fixed to get the ground state masses of D and D_s mesons. The meson radial wave function for $q\bar{q}$ combination is constructed with the respective quark and anti-quark wave functions given by Eqn. (5) and 6. The quark mass parameters m_c, $m_{u,d}$ and m_s are taken as 1.27 GeV, 0.37 GeV and 0.4 GeV respectively.

TABLE II: S-wave D_s ($c\bar{s}$) spectrum (in MeV).

nL	State	Present	Experiment		[6]	[7]
			Meson	Mass [1]		
1S	1^3S_1	2112.3	D_s^*	2112.3±0.5	-	2111
	1^1S_0	1970.6	D_s	1968.49±0.32	-	1969
2S	2^3S_1	2684.4	$D_{s1}(2710)$	2710^{+12}_{-7}	2728	2731
	2^1S_0	2603.9		2638 [9]	2656	2688
3S	3^3S_1	3195.1		-	3200	3242
	3^1S_0	3136.9		-	3140	3219
4S	4^3S_1	3676.0		-	-	3669
	4^1S_0	3629.3		-	-	3652
1P	1^3P_2	2572.3	$D_{s2}(2573)$	2571.9±0.8	-	2571
	1^3P_1	2433.7	$D_{s1}(2460)$	2459.6±0.6	-	2574
	1^3P_0	2341.3	$D_{s0}^*(2317)$	2317.8±0.6	-	2509
	1^1P_1	2420.4	$D_{s1}(2536)$	2535.12±0.13	-	2536
2P	2^3P_2	3023.2		-	3045	3142
	2^3P_1	2927.7		-	3020	3154
	2^3P_0	2864.1		-	2970	3054
	2^1P_1	2918.4	$D_{sJ}(3040)$	3044^{+30}_{-9}	3040	3067

Results and Discussion

The predicted S-wave masses of D and D_s mesons are in very good agreement with experimental [1] results as given in Table I and II respectively. The predicted results of P-wave D meson states, 1^3P_2 (2455.1 MeV) and 1^3P_0 (2276.6 MeV) are in good agreement with experimental results of 2462.6 ± 0.7 MeV and 2318 ± 29 MeV [1] respectively. The predicted results of P-wave D_s meson states 1^3P_2 (2572.3 MeV), 1^3P_1 (2433.7 MeV) and 1^3P_0 (2341.3 MeV) are also good found in good agreement with experimental results 2571.9 ± 0.8 MeV, 2459.6 ± 0.6 MeV and 2317.8 ± 0.6 MeV [1] respectively.

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